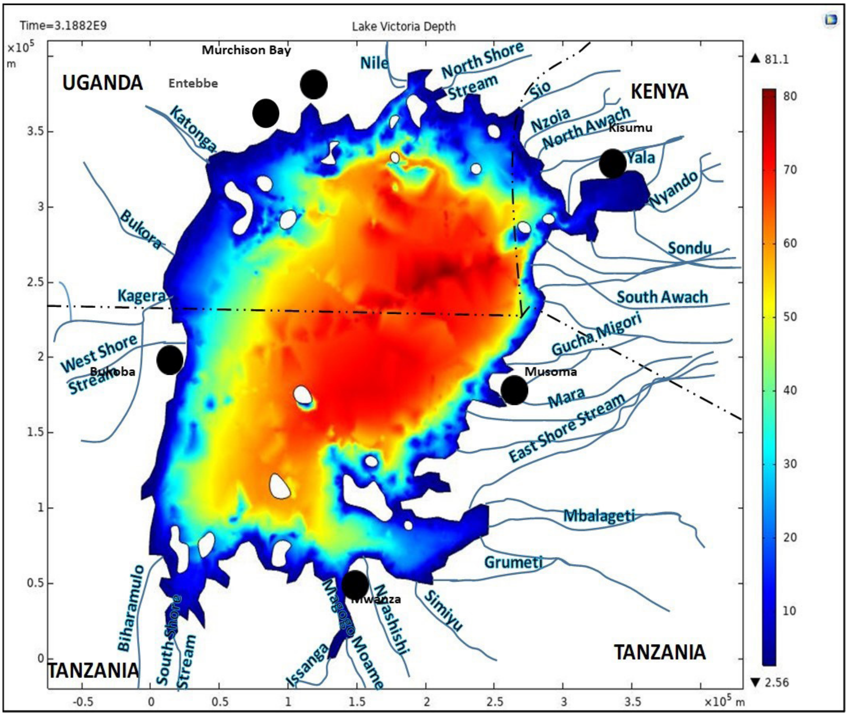
KTH ROYAL INSTITUTE
OF TECHNOLOGY

Doctoral Thesis in Land and Water Resources Engineering

Lake Hydrodynamics and Pollution Transport under Climate Change

The Case of Lake Victoria

SEEMA PAUL



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Academic Dissertation which, with due permission of the KTH Royal Institute of Technology, is submitted for public defence for the Degree of Doctor of Philosophy on Friday the 24th of November 2023, at 9:00 a.m. in D37, Lindstedtsvägen 9, Stockholm.

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Preface and dedication

To, my parents, family, and friends.

Ever grateful for your support, especially in finishing my work.

Water for all

Water is pure,
Water is natural,
Water is healthy,
Water can help all.

Water is simple,
Water is free,
Water can help the lives,
                    the lives of you and me.

— Olivia Taylor

On our part, we know this lake [Victoria] by a name infinitely more meaningful and beautiful. It is *Nalubaale*; which means 'Lake of Goddesses.' And the River Nile we call *Kiira*; which means the river which tumbles and bubbles out of *Nalubaale*.

— Paulo Wangoola in Williams et al., 2008.

Nalubaale… abode of the spirits.

— Peter Hoesing, 2012.

Abstract

A very small part of the total earth's water is freshwater (only 2.5 %). Unfortunately, due to climate change and pervasive manmade activities, surface freshwater quality in many places of the world has become degraded. This is manifested in the Rift Valley lakes, a series of lakes in Eastern Africa that runs from Ethiopia in the north to Malawi in the south. Water quality degradation in the Rift Valley lakes is driven by various factors, including water quantity and scarcity, pollution and contamination, nutrients loading, and general water use by industry and society. In particular, Lake Victoria, the world's second-largest freshwater body and the largest tropical lake, has seriously polluted near lakeshore areas, which is a great regional development problem causing misfortune for millions of people.

This dissertation contributes new insights into lake hydrodynamic processes and pollution transport in shallow lakes through developing more accurate models to understand the complex processes of water quality degradation. Based on empirical data this thesis developed systematic methods to consider lake bathymetry, lake flow, water level verification, water balance, hydro-climatological processes, transport and dispersion of pollutants and nutrient particles. The data-driven hydrological model of Lake Victoria that is developed in the thesis considers hydro-meteorological and climatological data, river discharges and outflow, wind speed and direction, atmospheric deposition, nutrient loading, concentration of pollutants and nutrients, and remote sensing satellite data. The thesis illustrates the power of numerical and hydrodynamic methods that uses one- and two-dimensional mathematical equations (1D and 2D) to model the three-dimensional (3D) behaviour of shallow lakes over time.

The results indicate that the lake hydrodynamics of Lake Victoria are heavily influenced by lake bathymetry and regional weather patterns and are thus connected to increasing climate variation. The hydro-meteorological processes, verified by empirical data on precipitation, lake flow and lake water levels, show that extreme weather events are responsible for changing the characteristics of lake water balance, changing seasonal variations, and exhibiting strong correlations among water level and hydro-meteorological data. The model of the movement of pollutants and nutrient particles shows how pollutants and nutrients travel within Lake Victoria and where they concentrate in the lake and its sediments. The wind hydrodynamic modelling shows that the wind, along with hydrodynamic stability, plays an important role in pollution flow patterns and that pollutants can be transported from shallow parts, when they leave rivers and shorelines, to deeper lake areas. The hydro-climatological model demonstrates the crucial interdependence between hydrodynamic processes and climatological factors at the catchment scale of Lake Victoria.

The numerical models and calculation methods that have been developed in this dissertation represent additional contributions to hydrodynamic research and can be used to investigate hydrodynamic processes in other lakes. The thesis contributes to UN Sustainable Development Goals related to water security, drinking water, food, and health. A potential area of application lies in supporting analysis and mitigation of pollution and climate change effects and more generally aid in the natural resource governance of this vital African lake.

Keywords

Bathymetry mapping; model verification; water balance; correlation; tracer transport; wind hydrodynamics; pollution and nutrient transport; climate effect.

Sammanfattning

En liten del av jordens vatten är sötvatten (endast 2,5 %). På grund av klimatförändringar som skapats genom ekonomiska och mänskliga aktiviteter har sötvattenskvaliteten på många platser i världen kraftigt försämrats. Detta märks i sprickdalssjöarna, en serie sjöar i området *Rift Valley* i östra Afrika som går från Etiopien i norr till Malawi i söder. Försämring av vattenkvaliteten i sprickdalssjöarna drivs av olika faktorer, inklusive vattenmängd och vattenbrist, spridning av föroreningar från jordbruk och städer, överbelastning av näringsämnen och överanvändning av vatten av industri och samhälle. I synnerhet Victoriasjön, världens andra största sötvattenkälla och den största tropiska sjön på Jorden, har fått allvarligt förorenade strandområden, vilket är ett stort regionalt utvecklingsproblem och orsakar problem för de miljontals människor som bor runt sjön eller som på olika sätt är beroende av dess vatten.

Denna avhandling bidrar med nya insikter om grunda sjöars hydrodynamiska processer och föroreningstransporter genom att utveckla mer exakta modeller för att förstå de komplexa hydrologiska processer som formar grunda sjöars vattenkvalitet. Baserat på empiriska data har denna avhandling utvecklat systematiska metoder för att beakta Victoriasjöns batymetri, sjöflöden, vattennivåer, vattenbalans och hur föroreningar och näringspartiklar transporteras och sprids i sjön. Den datadrivna flödesmodellen beaktade meteorologiska och hydrologiska data, vindhastighet och vindriktning, utflöden från floder, men också koncentrationen av spårämnen av föroreningar och näringsämnen. Avhandlingen illustrerar kraften i numeriska och hydrodynamiska metoder som använder en- och tvådimensionella matematiska ekvationer (1D och 2D) för att modellera det tredimensionella (3D) beteendet hos grunda sjöar över tid.

Resultaten indikerar att hydrodynamiken i Victoriasjön är starkt påverkad av batymetri och klimatförändringar. De hydrometeorologiska processerna, verifierade genom sjöflödes- och nivåmodeller, visar att extrema väderhändelser som troligtvis drivs av klimatförändringar är ansvariga för att förändra fundamentala egenskaper i sjöns vattenbalans och ändrade säsongsvariationer som uppvisar stark korrelation mellan vattennivå- och hydrometeorologiska data. Den modellerade transporten av spårämnen från utvalda floder, en modell för att förstå hur föroreningar och näringspartiklar sprids i sjön, visade hur ämnen färdas i Victoriasjöns vatten och var de över tid kan komma att koncentreras. Den numeriska hydrodynamiska modelleringen visar att vinden tillsammans med hydrodynamisk stabilitet spelar en viktig roll i hur föroreningar sprids i tydliga mönster och att föroreningar kan transporteras från grunda delar, när de lämnar floden eller stranden, till sjöns djupare delar och därmed deponeras och med tiden nå höga koncentrationer. Den hydroklimatologiska modellen visar vid skalan för avrinningsområdet på viktiga samband mellan hydrodynamiska processer och klimatologiska faktorer.

De numeriska modellerna och beräkningsmetoderna som har utvecklats i denna avhandling representerar bidrag till hydrodynamisk forskning. De metoder för att ta fram modeller för hydrodynamiska processer i sjöar, beräkna och verifiera sjöars batymetri, sjövattenflöde och föroreningstransport samt den uppsättning analytiska metoder som utvecklats parallellt med arbetet kan med fördel användas för att undersöka hydrodynamiska processer i andra sjöar. Ett potentiellt tillämpningsområde för denna forskning är att stödja analys av lokala och regionala effekter av klimatförändringar och föroreningar. Forskningen kan också användas för att utveckla strategier för att förhindra ytterligare föroreningar i sötvattensjöar och för att mer allmänt stödja arbetet med att förvalta denna centrala afrikanska sjö. Tillsammans bidrar avhandlingen med kunskap i linje med FN:s mål för hållbar utveckling relaterade till vattensäkerhet, dricksvatten, mat och hälsa.

Nyckelord: Batymetri kartläggning; modellverifiering; vattenbalans; korrelation; transport av spårämnen; vind hydrodynamik; föroreningstransport; klimateffekt.

Acknowledgements

This thesis was carried out at the Department of Sustainable Development, Environmental Sciences and Engineering (SEED) at Royal Institute of Technology, Stockholm between 2015 and 2022, as part of the Division of Strategic Sustainability Studies, and the Division of Land and Water Resources Engineering. I would like to express my deepest thanks to Prof. Henrik Ernstson at KTH for being my main supervisor and Associate Prof. Zahra Kalantari at KTH as my co-supervisor, and both for their solid support in finalizing my thesis. I also deeply thank my associated co-supervisory team: Prof. Jesper Oppelstrup (em.) at KTH; Prof. Steve W. Lyon at Ohio State University (USA); and Prof. Roger Thunvik (em.). I extend my thanks to Prof. Bo Olofsson (em.) for providing guidance in writing and analysis and I am grateful to Prof. Vladimir Cvetkovic at KTH who was my Licentiate supervisor. I thank all for giving me the opportunity of pursuing this PhD degree within the group of Sustainability Assessment and Management at KTH and I am grateful for their valuable support, guidance, and dedication during the development of this thesis.

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At the beginning, my work started as a collaboration between KTH and Makerere University. I was fortunate enough to visit Uganda and Kenya to gather foundational data from various organizations in those countries. I would like to extend a special thanks to Prof. Richard Wait (em.) at Uppsala University (Sida-ISP program) who was an active participant in this work from the very beginning. He helped us by providing the hydrograph map and initial lake map (Paper III) and assisted in collecting data from the field. I would also like to express my gratitude to Docent Bijan-Sam Dargahi (em.) at KTH, who provided us with the initial Lake Victoria velocity map of Delft 3D.

I wish to acknowledge Lake Victoria Basin Commission (LVBC) and Water Resources Management Authority (WRMA) for their useful cooperation. Most of the data described in papers I-IV and VI were obtained with the help of engineers Ramadan Omari and Reuben Ngessa at WRMA and LVBC, who provided more than fifty years of data for river inflow, outflow, precipitation, and evaporation, alongside the satellite data I collected myself. Thanks also to Tabu Munyalo, NaFIRRI, Uganda, providing us with extensive bathymetry dataset for the entire lake (used in paper I). I would like to express my gratitude to the National Water and Sewerage Corporation (NWSC) in Uganda for informing us about the water pollution issues in Murchison Bay. I am also thankful to Prof. Sally MacIntyre for providing us with information on the possibility of developing a method for transporting pollutants from shallower to deeper areas of the lake, which was our contribution in Paper IV.

Thanks to all our East African group partners for providing data from different sources (Paper I-VI). Special thanks to Tanzania Water Ministry and Tanzania Meteorological Agency, who provided more than thirty years monthly averages wind speeds and wind directions data (Paper IV). I am also thankful to the team of LVBC, WRMA, NWSC, NaFIRRI, who provided valuable datasets and collaboration from the very beginning.

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To my late Mother and late Father, thank you for making me who I am. This work effort is dedicated to both of you!

Stockholm, October 2023

Seema Paul

List of appended papers

Paper I

Paul, Seema, Jesper Oppelstrup, Roger Thunvik, John Mango Magero, David Ddumba Walakira, and Vladimir Cvetkovic (2019). Bathymetry development and flow analyses using two-dimensional numerical modelling approach for Lake Victoria. *International Journal of Fluids* 4: 1-21, https://doi.org/10.3390/fluids4040182.

Paper II

Paul, Seema, and Jesper Oppelstrup (2020). Hydro-meteorological processes driving solute transport in Lake Victoria. *Journal of Water Science* 34(1): 18-31, https://doi.org/10.1080/11104929.2020.1722416

Paper III

Paul, Seema, Benedict T.I. Reinardy, David Ddumba Walakira, Henrik Ernstson, Prosun Bhattacharya, and Zahra Kalantari. A shallow water numerical method for assessing impacts of hydrodynamics and nutrient transport processes on water quality values of Lake Victoria. *Heliyon* (Elsevier)(accepted with minor revision).

Paper IV

Paul, Seema, Jesper Oppelstrup, and Steve W. Lyon. Potential wind influences adopted for evaluating surface-water transport processes of Lake Victoria. *Journal of Hydrology* (under revision).

Paper V

Paul, Seema, and Jesper Oppelstrup (2020). Developed numerical simulation of falling and moving objects in viscous fluids under the action of a Reynolds Lubrication Theory and Low Reynolds numbers. *Open Journal of Fluid Dynamics* 10: 8-30, https://doi.org/10.4236/ojfd.2020.101002

Paper VI

Paul, Seema, Henrik Ernstson, John Mango Magero, Zahra Kalantari, and Steve W. Lyon. Understanding hydrodynamic and climate effects on the transboundary Lake Victoria basin at a catchment scale. (manuscript)

Author’s contribution to each paper

Paper I

The author SP and co-author JO were responsible for the scientific planning, theoretical development, modelling, and evaluation. The author was responsible for some of the field work and most of the writing. Comments and suggestions were received from the co-authors.

Paper II

SP and co-author JO were responsible for the scientific planning, theoretical development, modelling, and evaluation. The author was responsible for some of the field work and most of the writing. Comments and suggestions were received from the co-authors.

Paper III

SP and BR were responsible for the scientific planning. The approach for the data analysis were developed by SP and DDW. The whole paper design, structure and writing by SP, BR and HE, with feedback and co-writing from PH and ZK.

Paper IV

SP and co-authors jointly carried out the scientific planning, theoretical development, modelling, and evaluation. The author was responsible for some of the fieldwork, and the whole paper structure, design, and written by SP. Comments and suggestions were received from the co-authors.

Paper V

SP and co-author JO were responsible for the scientific planning, theoretical development, modelling, and evaluation. The paper written by SP.

Paper VI

SP and HE were responsible for the scientific planning, SP and JMM collected data from reliable sources, comments and suggestions were received from ZK and SWL. The author of this thesis was responsible for the theoretical development, modelling, and evaluation. The author was also responsible for collecting remote sensing data, fieldwork, and writing.

Other works not included in the thesis

- **Paul, S.**, Oppelstrup, J. (2022). Murchison Bay surface water pollution transport mechanisms. (manuscript)

- **Paul, S.**, Ijumulana, J. (2022). Land-use distribution criteria changing the groundwater contamination factor and transportation patterns at Mara River Basin of Lake Victoria. (manuscript)

- **Paul, S.** (2014) Hydrodynamics of Lake Victoria: Vertically integrated flow models in COMSOL Multiphysics. In: *COMSOL Conference 2014*, Bangalore, India, 13-14 November. The manuscript received the conference Best Student Poster award.

- Walakira, D.D., **Paul, S.**, Wait, R., Mango, J.M., Kasozi, J., Oppelstrup, J., Thunvik, R. (2014). Development of a 2D Vertically Integrated Shallow Water Equation Model of Lake Victoria. In: *33rd Conference of the Southern Africa Mathematical Sciences Association – SAMSA2014*, National University of Science and Technology, Victoria Falls, Zimbabwe, p. 107

- **Paul, S.**, Oppelstrup, J., Cvetkovic, V. (2018). Lake Victoria development model assessing by water level hydrodynamics, pollution transport and wind driven flow. *Proceedings of Twentieth National Symposium on Environment (NSE-20)*, IIT Gandhinagar, Gujarat, India, pp. 125-126.

- **Paul, S.**, Oppelstrup, J., Cvetkovic, V. (2019). Hydrodynamic wind induce model influencing inner Murchison Bay flow circulation. *GEET-19, International Conference on Green Energy and Environmental Technology*, Paris, France, pp. 61-66.

List of abbreviations

ALE	Arbitrary Lagrangian-Eulerian
CDF	Cumulative Distribution Function
CFD	Computational Fluid Dynamics
CFL	Courant-Friedrichs-Lewy
CM	Comsol Multiphysics software
CRU	Climate Research Unit
CRU TS	Climate Research Unit Gridded Time Series
DEM	Digital Elevation Model
DO	Dissolved Oxygen
DS	Dynamical System
E	East
EC	Electric Conductivity
ENSO	El Niño-Southern Oscillation
EOS	Earth Observing System
ET	Evapotranspiration
FEM	Finite Element Method
FFT	Fast Fourier Transform
FVM	Finite Volume Method
GIS	Geographical Information System
ITCZ	Inter-Tropical Convergence Zone
IWD	Inverse Distance Weighted
LV	Lake Victoria
LWB	Lake Water Balance
LVBC	Lake Victoria Basin Commission
LVEMP	Lake Victoria Environment Management Project
ODE	Ordinary Differential Equations

MB	Murchison Bay
MODIS	Moderate Resolution Imaging Spectroradiometer
N	North
NASA	National Aeronautics and Space Administration
NERC	Natural Environment Research Council
N-S	Navier-Stokes
NE	Northeast
NW	Northwest
NaFIRRI	National Fisheries Resource Research Institute
3D, 2D	Three, Two-dimensional
NOAA	U.S. National Oceanic and Atmospheric Administration
NWSC	National Water Sewage Cooperation
ORP	Oxidation-reduction potential
PDE	Partial Differential Equation
PET	Potential Evapotranspiration
POD	Proper Orthogonal Decomposition
POM	Princeton Ocean Model
POWER	Prediction of Worldwide Energy Resources
Re	Reynolds Number
RLT	Reynolds Lubrication Theory
ROMS	The Regional Oceanographic Model System
S	South
SE	Southeast
SW	Southwest
Sida-ISP	Sida, International Science program
SRTM	Shuttle Radar Topography Mission
SVD	Singular Value Decomposition

SWEs	Shallow Water Equations
TDS	Total dissolved solids
TIN	Triangulated Irregular Network
TVD	Total Variation Diminishing
USGS	U.S. Geological Survey
W	West
WLF	Water Level Fluctuation
WL	Water Level
WMO	World Meteorological Organization
WRMA	Water Resources Management Authority

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1. Introduction

Lakes are one of the most easily available and exploitable freshwater resources and they support a range of services for people and industry, from food production, irrigation, transportation, recreation, to hydropower. Yet, the surface and freshwater that exist in lakes, streams and wetlands make up only 0.01 % of the total global water budget (Sparrenbom et al., 2022). With an increasing world population and intensifying industrial activities, the pressure on accessing large volumes of high quality water from the world’s largest lakes is building (**Figure 1a**). Over the last decades, climate change has furthermore emerged as yet an additional and uncertain stress factor impacting quality and quantity of available lake water resources (Papa et al., 2023). Taken together, lakes have become an increasingly valued resource for society and requires management and protection (Schewe et al. 2014; ECO-Rep Training Manual). In this context, to better understand lake hydrodynamic processes is crucial for sustainable development and environmental water resource management where the study of hydrodynamics and how human activities impact lake water quality is an important yet poorly understood issue, further complicated by how hydrodynamics can vary greatly between different lakes because of geometric differences, surrounding topographic, hydrological, and geochemical loadings, and meteorological exposures (Liu et al., 2021; McGinnis and Wuest 2005).

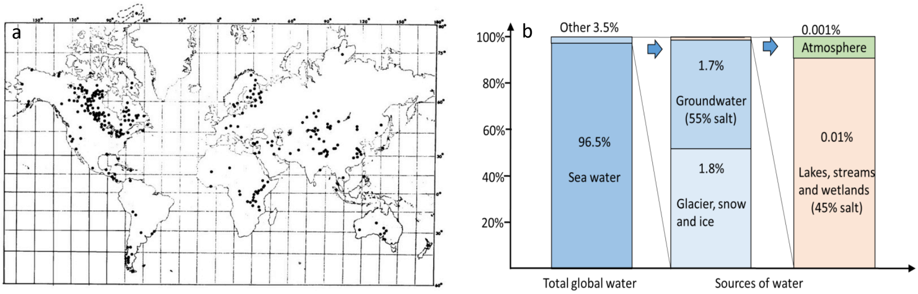

Figure 1: The global largest lakes water distribution: a) worldwide largest lakes (source: Charles, 1982); b) the total global freshwater distribution as well as water in lakes, courtesy of Prof. Bo Olofsson (source: Sparrenbom et al., 2022).

This dissertation contributes new insights into lake hydrodynamic processes and pollution transport in shallow lakes through a focus on Lake Victoria, also known by its indigenous names as *Nyanza*, *Nam Lolwe*, and *Nalubaale*—“the abode of the spirits”[1]. It is the world’s third-largest lake, second-largest freshwater lake, and largest tropical lake (Paul et al., 2019) and lies in a distinguished climate and biometric region of Earth where tropical temperature remains relatively constant throughout the year and seasonal variations are dominated by precipitation (Christopher et al., 2014; Hasan & Jin, 2014; Kundu et al., 2017). Lake Victoria is indeed vital for its region and lies across three East African countries: Kenya with 6 %, Tanzania with just over half at 51 %, and Uganda extending 43 % across the lake’s surface area (Hassan and Jin, 2014; Christopher et al.,

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[1] British colonial explorer John Hanning Speke named the lake after Queen Victoria in 1858. But it obviously had other names before European colonization. On its southern shore it was named *Nyanza*, a large body of water; on its eastern shore the Luo called it *Nam Lolwe*, the endless lake; and in the north the Lugandan people called it *Nalubaale*, a powerful deity and the mother of all other gods, “the abode of the spirits” (Hoesing, 2012: 100; see also Williams et al. 2008).

2014; Kundu et al., 2017; Paul et al., 2019) and measures 355 km across its widest surface area. Lake Victoria is considered a shallow lake with an average depth of 40 meters and a maximum depth of 80 meters (Kayombo and Jorgensen, 2005; Cheruiyot and Muhandiki, 2014; Paul et al., 2019) and it is the major freshwater source for domestic, agriculture, industrial, fishery, and transport purposes in the surrounding region (LVFO, 2015; Kundu et al., 2017) providing resources for livelihoods and ecosystem services for over 40 million people (Kundu et al., 2017). The Lake Victoria Basin's population has grown from 35 million in 2006 to 45 million in 2017 (World Bank, 2018) and the basin has undergone large Water Level (WL) changes caused by multiple factors such as heavy rainfall, human activities, shifts in lake bottom topography, environmental degradation, urbanization, and climate change (Awange et al., 2008; Akurut et al., 2014). The economy of the riparian areas are heavily dependent on Lake Victoria with over 80 % of the basin population relying on agriculture and livestock activities for their livelihood that generates 30 – 40 % of the regional GDP (World Bank, 2018). The gross economic product of the lake catchment is on the order of US$ 400 – 500 million annually and the lake catchment supports nearly one-third of the total population of East Africa (Kayombo and Jorgensen, 2005; World Bank, 2018). Regular monitoring of lake water level is crucial because how the region's economy and access to water, food, and transport depends on the lake (Semazzi, 2011).

The lake has furthermore a long shoreline with numerous gulfs and bays, many of which are recipients of river discharge, municipal and industrial effluents (Gikuma-Njuru and Hecky, 2005; Cornelissen et al., 2014; MacIntyre et al., 2014). Hydrodynamic flows in lakeshore areas are furthermore complex due to variations in morphometry that affect water, sediments loads, concentrations of chemicals, and ecological diversity (Luyiga et al., 2015; Gikuma-Njuru et al., 2018). Some rivers carry industrial discharges directly to the shoreline. An example is the shallow Winam Gulf in Kenya, near Kisumu in the east of the lake (Kyomuhendo, 2003; Longgen et al., 2009; Kundu et al., 2017). Malfunctioning sewage plants discharge inadequately treated sewage into tributary rivers and subsequently into the lake. There are also examples where agricultural and chemical industries discharge pollutants directly into the lake (Christopher et al., 2014). In Tanzania small-scale gold mining activities bring significant pollution in the form of heavy metals (LVEMP, 2005); in Jinja, Uganda, food processing, textile, leather, paper production and metallurgy industries brings further pollution (Oguttu et al., 2008); and pollution is also directly drained from Nakivubo channel into Murchison Bay affect near-shore areas (Kabenge and Wang., 2016). These activities from the three countries severely affect lake water quality and are continuously degrading the water even in deeper lake areas (Scheren et al., 2000; Joachim, 2011). The most important components of water quality degradation in Lake Victoria are due to pollution, river discharge, diffusion processes, nutrient loading, eutrophication, and the spread of the invasive South American water hyacinth (waterweed)(Kabenge and Wang, 2016; Olokotum et al., 2021; Akurut et al., 2017; Odhiambo 2016; Banadda et al., 2011; Lubovich, 2009).

During the past 70 years, Lake Victoria has been seen as an "international water body" that is under considerable pressure from a variety of interlinked human activities (Odada et al., 2004; Hazenoot, 2012). The current and perhaps most alarming anthropogenic stress to be considered is the increasing demand for hydropower which puts most pressure on the lake during the drought seasons (or drought years), a pressure expected to increase (Mutenyo, 2009; Hecky et al., 2010). Recent studies on climate variability and fluctuations of lake levels show some worrying drought patterns (Awange et al., 2008; Awange et al., 2013; Sewagudde, 2009; Swenson and Wahr, 2009; Yin and Nicholson, 1998). The important factors are precipitation, temperature, evaporation, drought, hydro-meteorological processes, and floods which all significantly impact the lake water levels with consequences for the lake water quality and surrounding people's livelihoods (Khaki and Awange, 2021; Paul and Oppelstrup, 2020; Nkuna and Odiyo, 2000). However, the low water quality brings poor health (Machiwa 2003), lowers livelihood opportunities, creates gender inequality, causes absence of children from school, increases domestic expenditure on health, and imbalance ecosystems (Khaki and Awange, 2021). All these consequences ultimately deepen the poverty cycle for the surrounding countries' people and pose significant challenges to many of the sustainable development goals (in particular SDGs 1-6, 8, 10, 13, and 14) (Koutsouris et al., 2010). Therefore, it's crucial to monitor and manage the basin's hydrologic cycle using up-to-date technology and methods (Awange et al., 2013).

1.1. State of the art

Given the centrality of Lake Victoria as a regional resource and its many challenges in terms of pollution and climate variability, this thesis focuses on developing a cutting-edge and future-looking hydrological model of Lake Victoria that can use and is validated by existing and new empirical data. Despite Lake Victoria's importance for economic activities, agriculture, and livelihoods, it has not been adequately studied in terms of hydrodynamic changes (Gronewold et al., 2020; Awange et al., 2019). There is a great need to better understand how the lake, as a dynamic system, is impacted by biophysical and climatic conditions, as well as anthropogenic and economic activities. This involves creating hydrodynamic process models that considers the lake's irregular boundaries and bathymetry, and weather and climatic patterns of its surrounding catchment area to study factors such as lake water levels, pollution transportation, eutrophication, and water quality (Sangale et al., 2005). According to Gronewold et al. (2020) and Awange et al. (2019), the lake has not received enough attention and research, which makes it crucial to conduct more studies to develop a better understanding of this natural resource.

Lake Victoria has complex water systems due to a variety of factors. Firstly, the lake is environmentally sensitive and has a shallow water body system, making it challenging to manage its flow (Pacini & Covelli, 2002; Curtarelli et al., 2014). Water level fluctuations (WLFs) could also have massive implications for local communities (Semazzi, 2011) and influence the outflow released from Lake Victoria, with consequences for hydropower generation and energy availability in the region (Vanderkelen et al., 2018). Furthermore, the modelling of pollutant and nutrient dispersion in shallow lakes is important to help with monitoring and management. Additionally, climate change and the increasing frequency of weather extremes have made shallow water bodies such as Lake Victoria more vulnerable to wind, evaporation, and human interference compared to deep lakes (Gulati et al., 2008; Leira & Cantonati, 2008; Nutz et al., 2018). Lake Victoria and surrounds suffers currently from a severe and prolonged drought that has imposed additional pressure on water availability, accessibility, and demand in the basin (LVBC, 2018). Collectively, hydro-meteorological processes determine the water level in the lake and frequent extreme events create surface and sub-surface runoff, which contribute to sources of pollution and sedimentation in the lake (Okungu et al., 2014). Taken together, there are several reasons showing the necessity of developing hydrological models for shallow lakes of which Lake Victoria serves as an important case study to develop techniques to handle long-term data series and try out various mathematical modelling approaches.

Hydrodynamic models of Lake Victoria exist, including models based on thermodynamic and hydrodynamic characteristics from the Princeton Ocean Model (POM) 3D simulations (Song et al., 2003; Anyah and Semazzi, 2004; Anyah et al., 2006; Anyah and Semazzi, 2009). For simplicity, however, these older models never considered the complex shoreline nor the real bathymetry of the lake but assumed an elliptic lake with surface wind stress to investigate the vertical lake temperature profiles. Nyamweya et al. (2016) in turn used the Regional Oceanographic Model System (ROMS) to investigate how temperature and currents in Lake Victoria effect the diurnal, seasonal, and annual variations in stratification, vertical mixing, and inshore-offshore exchanges in the water column. USA's National Oceanic and Atmospheric Administration (NOAA) contributed a model for Lake Victoria with meteorological data as the first part of an integrated physical and ecological model with a focus on eutrophication (Damsgaard, 2012). Water level, pollution, eutrophication, and sediment flow analysis were partly also covered by Hecky and Kendall (LVEMP, 2005; Kendall, 2016), but an elaborate bathymetry mapping combined with detailed hydrodynamics and analysis of climate effects on the dynamic processes that affect the lake's hydrology has previously not existed, which is what this thesis set out to develop.

The starting point for this study is furthermore grounded in recommendations made by influential regional and international policy- and research bodies, including the Lake Victoria Basin Commission, LVBC (Mwanuzi et al., 2005), including the LVBC Climate Change Adaptation Strategy and Action Plan, 2018-2023 (LVBC, 2018), the LVBC Operational Plan 2015-2020 (USAID, 2015), and the Lake Victoria Environmental Management Project III (World Bank, 2018). A key recommendation across these reports was to develop a detailed lake bathymetry mapping for hydrodynamic investigations, which is crucial for future water quality assessment and lake level management.

In addition, there are no models of Lake Victoria that explicitly takes into account wind stress, which is a key mechanism of shallow lakes that will affect lake flow and wave motion and promote and shape pollution transport along the lake (Oporto et al. 2016; Wu et al., 2010). The spatial variability of the wind stress strongly influences circulations, transport, and mixing processes (Torma & Kramer, 2016). The spatial variability of the wind stress in Lake Victoria affects not only the surface layer circulation but also the depth-averaged flow field (Podsetchine & Schernewski, 1999; Jozsa, 2001). The wind-induced hydrodynamic as well as the hydrostatic stability could transform flow patterns in the lake and therefore must be incorporated into numerical pollution transport models. The study of Lake Victoria in this PhD thesis becomes important also in this regard.

More precisely, what is lacking in previous studies are explicit consideration of a detailed lake bathymetry mapping together with hydrodynamic investigations. To address this shortcoming, this dissertation considered a detailed bathymetry mapping together with a vertically integrated Shallow Water Equations (SWEs) model for lake hydrodynamics. The dissertation analyses the water dynamics of the lake including the transport and dispersion of pollutants and nutrients by using observed and remote sensing data. The thesis also considered three Tanzanian weather datasets (Bukoba, Mwanza, and Musoma) and one Uganda weather dataset to assess wind patterns and water flow trends. Further, a developed depth-resolved hydrostatic model was used to explore transport process for Murchison Bay (MB), a highly polluted bay in Lake Victoria close to Kampala, to understand pollution transport processes and characterize the exchange relationship between MB and the greater Lake Victoria. The thesis also draws upon extensive environmental data sets to understand how climate change is affecting Lake Victoria and its vast catchment areas.

Collectively, the framework developed in this dissertation contributes to the literature (reviewed briefly above and more extensively in the individual papers) with a new approach to help isolate the relative impact of the various hydrodynamic processes on the overall water quality of Lake Victoria. The results of the thesis can also be used to develop similar hydrodynamic models for other shallow lakes in the world. The hydrological model developed in this thesis furthermore contributes a tool that, with time, could be used to improve the management of water quality of Lake Victoria on which more than 40 million people depend.

1.2. Aim and objectives

This study aimed to gain a deeper understanding of the primary flow to pollution transport processes and climate change influencing hydrodynamic processes in Lake Victoria. The thesis drew on the perspective of shallow lake hydrodynamics and developed six papers (**I-VI**). These papers aimed to map the bathymetry of Lake Victoria, analyse hydro-meteorological processes, tracer transport, dispersion of pollutants and nutrients of lake water system, and studied the small movement of microorganisms in water towards modelling biological pollution. The thesis also created models to study how wind patterns affect lake hydrodynamics and pollution transport processes. Additionally, it mapped the effects of climate variability on Lake Victoria's dynamic processes.

The thesis main objective was broken up into several aims across the six papers. In paper **I** the main research aim was to develop a lake bathymetric mapping from generated and existing raw data and connect it with a lake flow model. Papers **I-II** focused on developing knowledge of shallow lake hydrodynamic behaviour under water stress and how hydrodynamic flow behaviour influences tracer and pollution transport. In papers **I-VI** the main research aim was to analyse which factors, including geodynamic, bathymetric, land use, tributaries, wind, and climate variation, can help understand hydrodynamic behaviour under climate stress and which factors are the most important for influencing pollution transport in shallow lakes. The resultant hydrostatic stability model (Paper **IV**) provides a base for lake water quality investigations that can help support the development and evolution of water quality management in Lake Victoria and its catchment area. Since the model takes into account pollution and eutrophication (Papers **III** and Paper **VI**) it could play a crucial role for natural resource management and sustainable development in the region. While not analysed in detail here, this dissertation provides a hydrodynamic scientific base for comprehensive lake management and regional water governance.

Table 1: Specific objectives for each paper and general contribution to shallow lake hydrodynamics.

No.	Specific objective of the thesis	General contribution to shallow lake hydrodynamic research
Paper I	To develop a systematic method for lake bathymetry mapping and analyse 2D flow behaviour verified by outflow and water level data for Lake Victoria.	Develops systematic methods for analysis of shallow lake bathymetries. The developed bathymetry is used in a numerical hydrodynamic model validated by outflow and water levels data. Provides an important comparative case study with other shallow lakes.
Paper II	To observe the relevant hydro-meteorological processes using monitoring data in the lake basin and use model simulations to assess whether possible interventions in the lake will affect pollution and tracer transport in the lake.	Employs the numerical model developed in Paper I to evaluate the lake water balance and seasonal lake flows. The paper shows how shallow lake models can be driven by hydro-meteorological data and describes the vertically integrated circulation and the soluble contents transportation.
Paper III	To assess how hydrodynamics, including wind stress, and nutrient transport processes influence water quality of Lake Victoria.	Selects the SWEs as a primary test problem for different numerical methods of Partial Differential Equations (PDE) including influence of wind speeds and a phosphorus deposition sub-model to calculate

		atmospheric deposition for water quality assessment.
Paper IV	To develop models that describe the wind hydrodynamics on lake catchment and the Murchison Bay as well as to increase the knowledge of possible movement of pollution from Murchison Bay to greater Lake Victoria.	Considers Lake Victoria and Murchison Bay near shore phenomena extended by wind hydrodynamics and transformation of pollution transport. The wind driven flow demonstrated a 'typical' wind flow and the depth-resolved hydrostatic model was used for transport process stabilization.
Paper V	To develop a model for a small object moving in polluted water.	Develops a general case and numerical simulations of how small organic particles travel in a viscous fluid with wave motion (a small wiggling worm or microorganism) towards modelling organic pollution.
Paper VI	To develop an understanding of lake hydrodynamics and the potential climate impact on Lake Victoria	Discusses how hydrodynamic processes in the catchment area of Lake Victoria are affected by climate change and how that influence the lake.

1.3. Scope of work

The thesis has covered the research knowledge gap of shallow lake hydrodynamics through numerical modelling using real data, remote sensing data and realistic synthetic data. The methodologies developed here are general and may hence be applied elsewhere. Most conclusions are applicable to apply in bathymetry development and shallow lake hydrodynamics. **Figure 2** shows the relationships between the works described in Papers **I-VI** in this thesis.

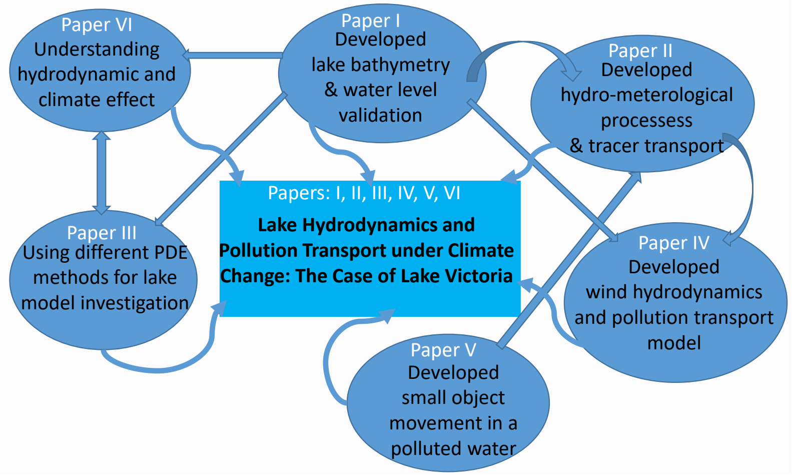

Figure 2: A schematic overview showing the procedures considered in this thesis (papers I-VI) to understand lake hydrodynamics and how bathymetry and lake dynamical processes interact.

1.4. Thesis structure

The thesis is organized into one longer essay (the *kappa* in Swedish) with six chapters followed by six papers in an appendix. Introductory **Chapter 1** explains the background, the state of the art of this research work, the objectives of the study, and the content of the thesis. **Chapter 2** describes the study area and materials and data used for modelling. **Chapter 3** describes methods used, focusing on the systematic processes that makes it possible to take into account lake bathymetry for developing lake hydrodynamic models using the Comsol software package.[2] This includes a review of detailed numerical methods and tools that has been used, including how geospatial data, lake levels, and shoreline data (section 3.2), alongside movements of small wiggling object and wind flow patterns (section 3.3) are used in developing numerical hydrostatic stability models and pollution transportation (detail descriptions of methods are found in the individual papers). **Chapter 4** summarizes the novel contribution of this thesis, as the results in **Papers I-VI** are presented. **Chapter 5** discusses the main results of the individual papers. **Chapter 6** provides the main conclusions, pointing out study limitations and future research. The six papers on which the thesis is based are attached in the appendix.

2. Materials: Detailed study areas and data

The thesis builds its models and analysis on a range of empirical data gathered from various sources including: LVBC, WRMA, NaFIRRI, Tanzania Water Ministry and Tanzania Meteorological Agency, synthetized real data from different hydrographs, and remote sensing data from different sources (**Table 2**). Below some details about this data is provided alongside distinct background features of Lake Victoria. For details see individual papers.

2.1. Basin scales with data, Paper I-VI

Lake Victoria is located between the Eastern and Western Branches of the East African Rift System in an elevated plateau at an altitude of $1135\ m$ in the western part of Africa's Great Rift Valley. The lake has a wide land catchment area, and the lake is close to rectangular with a total catchment area of 251,000 km^2, surface area of approximately 68,800 km^2 and a water volume at 2,760 km^3 (Kayombo and Jorgensen, 2005). Its shoreline is very irregular, with a total length of $30{,}000\ km$ (NBS, 2002). The northeastern part of the lake catchment is relatively steep and forested, while the southeastern part is drier and flatter (Lipzig and Thlery, 2017). The lake stretches $412\ km$ from north to south, between latitudes $0031'N$ and $3012'S$ and $355\ km$ from west to east between longitudes $31037'W$ and $34053'E$. It occupies a shallow depression ($80\ m$ deep) with many small islands and numerous streams. The surrounding areas contain different environments and a total of twenty-three rivers delivering significant inflow (Figure 3b). The largest contributing river, the Kagera, originates from the southwestern part of the mountains of Rwanda and Burundi. The Victoria Nile is the only outflow, situated in the city of Jinja in Uganda. The Nalubaale Dam complex (Lipzig and Thlery, 2017) essentially regulates lake water levels through controlling outflow (Figure 3a).

Lake water balance is determined by tributary inflow, outflow, rain over the lake and evaporation (Vanderkelen et al., 2018). Outflow and water level are easily measured, but tributary inflows, precipitation and evaporation are difficult to measure or estimate (Sene et al., 2018). The major contributor to water level variability is rainfall on the lake. The El Nino-Southern Oscillation

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[2] COMSOL Multiphysics is a trademarked modelling package for the simulation of any physical process that can be described with partial differential equations (PDEs). KTH has a software license that was used for this study. Read more here: https://www.comsol.com/

(ENSO) dominates the East African rainfall distributed over the lake by the movement of the Inter-Tropical Convergence Zone (ITCZ) (Smith and Semazzi, 2014; Kite, 1982). Rain is measured on metrological stations onshore and extrapolation to rain over the lake itself adds uncertainty (Thiery et al., 2015; Sene et al., 2018). Evaporation is very sensitive to air humidity on the lake and near-shore surface (Kite, 1981; Piper et al., 1986; Sene and Plinston, 1994; Tate et al., 2004; Smith and Semazzi, 2014), while the air temperature is nearly constant, so it is of minor influence in evaporation variation. However, Lake Victoria and its catchment vegetation play an important role to change evapotranspiration rate and to moderate the climate impact through contribution to precipitation, temperature, humidity, plant cover, spatial variability of drought, soil-water management system and yet other factors. The Water Resources Management Authority (WRMA) and Lake Victoria Basin Commission (LVBC) provided more than fifty years of precipitation, evaporation, in- and outflow daily averages data for the study. The LVBC and WRMA data differed slightly from each other, 3 % for lake inflow and evaporation, and 9 % for precipitation.

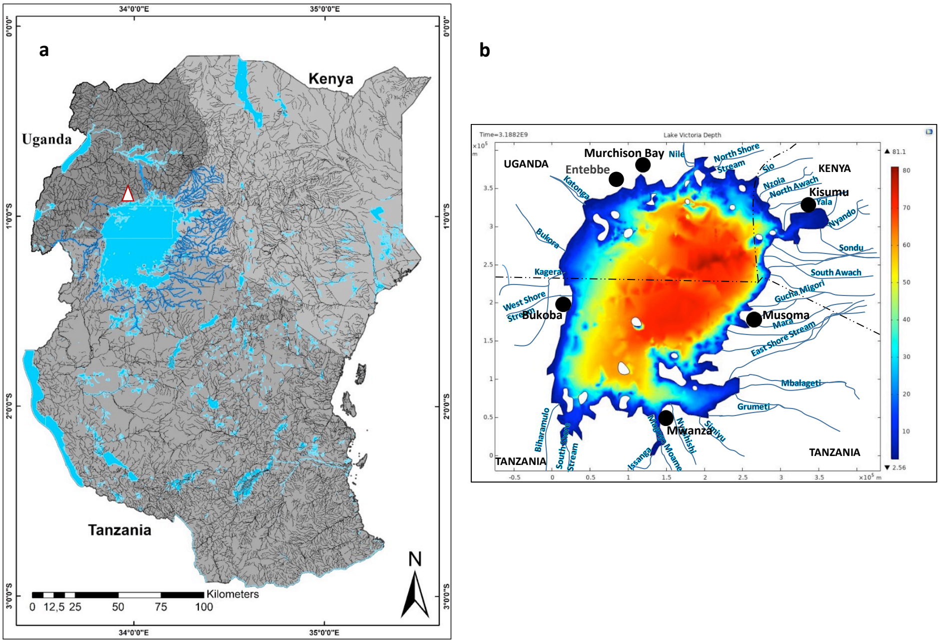

Figure 3: Lake Victoria basin maps with coordinate systems: a) Lake Victoria surrounding basins scale with Murchison Bay marked by a red triangle and all water basins (blue colour) together with numerous stream flows (black lines); b) Lake Victoria basin with depth scale bar and all river inflows (marked by blue lines) together with weather stations and climatology data stations used for this thesis (marked by black circles).

2.2. Geospatial data, Paper VI

One hundred twenty years of climatological data, including yearly and monthly rainfall and temperature data, have been downloaded from different remote sensing sources. Multi-spectral satellite images with lake depth, hydro-meteorological data, and remote sensing data were employed to study lake hydrodynamic and climatology. DIVA-GIS provided free spatial data for the whole world, which was used to visualize basin areas with all lake streams.[3] The Climate Research Unit (CRU) TS dataset consists of a long record of precipitation, temperature, drought, and clouds from 1901 to 2020 and uses a complex interpolation method to fill in gaps between nearest station data.[4] Freely downloaded global temperature and precipitation monthly time-series data (1901 to 2020) from a Google Earth-based interface to the CRU TS dataset were used to scientifically improve understanding of the climate system and its interactions with society.[5] Distributed yearly time series (1901 to 2020 data) visualized the monthly linear rainfall trend of the lake based on five near-shore weather measurement stations spread around the lake in the west, north, east, and south in Bukoba, Entebbe, Kisumu, Musoma, and Mwanza, respectively (Figure 3b, black dotted). More than 40 years of monthly and annual average temperature data was used from the NASA/POWER (Prediction of Worldwide Energy Resources) Data Access Viewer to visualize the actual evapotranspiration rate.[6]

2.3. Uganda, Tanzania, and Kenya climatology data, Paper IV and VI

Thirty years of monthly average wind speeds and wind directions data were received from Tanzania Water Ministry and Tanzania Meteorological Agency and used in **paper IV**. Tanzania controls the largest share of Lake Victoria with 51 % of the total area with catchment areas being Musoma, Mwanza and finally Bukoba that lies near the largest inflow at Kagera river (Scheren et al., 1994) (**Figure 3b, black dotted**). Tanzania has a tropical equatorial type of climate with great biological diversity due in part to the country's diverse topography. While Tanzania is characterized by two rainfall regimes, namely unimodal and bimodal rainfall regimes, Lake Victoria has bimodal rainfall regimes (Timiza, 2011). The bimodal rainfall regime has two rainy seasons, the heavy rainfall season (Masika) experienced between March and May and the short rainfall season (Vuli) occurring between October and December. Each rainy season is separated by a dry spell from June to September, which is sometimes interrupted by occasional drizzle (Timiza, 2011; Tungaraza et al., 2012). From 1922 to 2005, the annual rainfall was the highest along the western margin (Bukoba, 2.2 m/y) and the lowest was in the southeast (Musoma and Mwanza, $0.9 - 1.2\ m/y$) (Tungaraza et al., 2012; MacIntyre et al., 2014). The meteorology over the lake is structured by the presence or absence of the southern winds from the Indian Ocean that generally begin in May and end in August (MacIntyre et al., 2014).

2.4. Murchison Bay wind data, Paper IV

The Murchison Bay covers about 30 km^2 and lies in the northern part of Lake Victoria in Uganda and runs from Kampala to Entebbe (**Figure 3a, red triangle**). The Murchison Bay has a temperature ranging from 23℃ to 32℃ (Campbell, 2001). The climate is tropical with small variation throughout the year. The mean annual rainfall is about 1180 mm with two peaks in March to May and October to November (Matagi, 2002). The Murchison Bay has an Inner Murchison Bay (IMB) separated from the Outer Murchison Bay (OMB) that is situated at Ggaba 4.8 km from the mouth of Nakivubo channel. The bay is shallow with a gently sloping bed to a depth of about 11 m to 12 m in the Ggaba areas and about 22 m to 30 m at Entebbe. The MB covers about 18 km^2 with a mean depth of 3.2 m, but the whole watershed for the MB is 282 km^2 and the Nakivubo sub

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[3] https://www.diva-gis.org/

[4] https://www.uea.ac.uk/groups-and-centres/climatic-research-unit

[5] https://www.uea.ac.uk/web/groups-and-centres/climatic-research-unit/data

[6] https://modis.gsfc.nasa.gov/data/

catchment represents only about 17 % of the total watershed area yet it contributes 75 % of the effluent. Direct urban runoff enters the lake through the Nakivubo channel, which widens from about 5 m to 20 m, spanning a length of about 10 km. The Makerere hill, one of the highest points of Kampala city, slopes down to the Nakivubo wetland to efficiently drain the city centre of floods, which has increased the concerns for the water quality in the bay (Luyiga et al., 2015). The thesis explores the demand of National Water Sewage Cooperation (NWSC, Kampala, Uganda) because most of the wastewater are discharging into the lake through the Nakivubo channel of Murchison Bay, and the Bay lakeshore is over-polluted.

To build a $2D$ vertically integrated shallow water equations model in Comsol, the coefficient form of the partial differential equations (PDE) module was applied. We then developed a Matlab function $z(x, y)$ to create Comsol grid points and then to import those grid points into Comsol.

Table 2: Summary of the different sources of data used in this thesis.

Data sources	**Basin areas**	**Require data/method**	**Model descriptions**	**Papers**
WRMA, LVBC	Lake Victoria Basin	51 years of precipitation, evaporation, inflows, outflow data	Lake flow behaviours, verified lake's level, solute transport, wind hydrodynamics etc.	**I-VI**
Digitized data, NaFIRRI	Lake Victoria Basin	Bathymetry raw data	Processing, setting and developed bathymetry model for lake hydrodynamics	**I-VI**
DIVA-GIS	Surrounding Basin areas	Remote sensing data	Administrative areas with water resources	**III, VI**
CRU TS data	Lake Victoria Basin	120 years remote sensing rainfall, temperature, cloud, drought data	Observation climate scenarios on Lake Victoria	**VI**
Google Earth-based interface to the CRU TS dataset	Bukoba, Mwanza, Musoma, Kisumu, Entebbe	120 years remote sensing rainfall and temperature data	Potential climatographic and long-term seasonal rainfall	**VI**
NASA/POWER Data Access Viewer	Surrounding lake basins	Forty years of Remote sensing, evapotranspiration and temperature data	Surrounding basin evapotranspiration affect and flow rate	**VI**

Tanzania Water Ministry and Tanzania Meteorological Agency	Bukoba, Mwanza, Musoma	Thirty years of monthly averages wind speeds and wind directions data	Wind hydrodynamics and wind flow trends	**IV**

3. Methods

The modelling of this thesis have been implemented in Comsol Multiphysics (CM) software. An advantage of CM is that it makes possible the analysis of systems that have component parts with dissimilar physics so that for instance wind, water and climate dynamics can be interlinked numerically within the same modelling framework. This makes it relatively easy to analyse and develop models that contain multi-physics features. Learning to use this ability of CM, this thesis has improved the understanding of the linkages between lake bathymetry, hydrodynamics process and climate effects. The foundational model developed in **paper I** is the vertically integrated 2D shallow water equation model, SWE, which was chosen based on the assumption that Lake Victoria is considered a shallow lake where the average depth to surface length ration is very low ($H/L \ll 1$; see **section 3.1.2**). The lake hydrodynamic processes and forces that have been taken into account in developing the hydrological model are as follows: integrating shoreline and bathymetry, water sources and fluxes, water level, hydro-meteorological interactions (including extreme events frequency), tracer transport, wind driven hydrodynamics and pollution transport, microorganism in polluted water, and hydro-climate processes.

The development of the lake hydrodynamic model is explained below in three sections: the methods used for developing the lake bathymetry (**section 3.1**); different methods used for arriving at a hydrodynamic model (**section 3.2**); methods for analysing the movement of small objects, wind hydrodynamics, and pollution transport and dispersion (**section 3.3**). **Table 3** provides and effective summary of these methods including data acquisition, data analysis and study objects. A detailed description of the methodology for each study can be found in **Papers I-VI**.

3.1. Lake bathymetry development methods

The lake surface elevation measurement is likely the most fundamental property for lake hydrodynamic models. This measurement is relevant for a wide range of water quality resources and societal needs. The exact water depth or the morphology of the lake bottom topography is vital but remains uncertain in many parts of the world's lakes, rivers, and oceans (Hell, 2011). This thesis covered bathymetry mapping development in relationship with the lake hydrodynamics. This consisted of merging all raw data of lake bathymetry to filter and interpolate into a Digital Elevation Model (DEM) system that that was imported into the CM simulating software.

Table 3: The three method sections summarized (sections 3.1-3.3).

Study no	**Aim of the study**	**Study objects**	**Data collection**	**Data analysis**	**Paper**
1	To develop lake geometry, data-driven flow behaviour, and model verification	A case study of Lake Victoria, total catchment area which is 251,000 km^2, the lake surface area is approximately 68,800 km^2	LVBC, WRMA, Admiral Bathymetry, NaFIRRI: Digitized iso-depth curve data, Lake bathymetry raw data and 51 years of precipitation, evaporation, inflows and outflow	Several methods developed from several sources for lake bathymetry mapping, vertically integrated Saint–Venant shallow water equations (SWE) applied for lake flow behaviour and verified model in Comsol Multiphysics software by lake level and outflow.	**I**
2	To understand hydro-meteorological processes, correlation, and solute transport	A case study in Lake Victoria and its extreme events	Detailed analysis of LVBC and WRMA daily averages data of precipitation, evaporation, 23 river inflows, and outflow with initial tracer concentration	Hydro-meteorological processes, lake water balance, correlation lake outflow and water level, extreme events, transport of diluted species developed by advection-dispersion process.	**II**
3	To integrate SWE with different methods of PDE and atmospheric deposition to achieve a suitable solution for lake water system	A case study in whole lake catchment	LVBC rainfall data and mass balance data	SWEs model integrated with schemes and methods, and site-specific atmospheric deposition	**III**

4	To understand wind flow patterns and its trends and its effects on the pollution transport process	A case study in Lake Victoria, Murchison Bay, Tanzanian three weather stations and one Uganda weather station.	Wind-patterns tools, wind-driven flow model, depth-resolved hydrostatic stability, hydrodynamic transport process	Identify wind flow patterns and its trends; wind speed simulation; discovered the depth-resolved numerical hydrostatic stability required artificial dissipation for pollution transport.	**IV**
5	To understand locomotion of microorganism	A numerical case study considered	Developed a system for a thin two-dimensional (2D) worm-like object wiggle that is passing a wave along its centerline and its motion by ODE and moving mesh (ALE) technology.	The developed method's result shows that it is possible for the small object to have a motion from one position to another through small amplitudes and wavelengths in viscous fluid.	**V**
6	To understand the effect of climate change and hydro-meteorological process	A case study of a total lake basin surrounded by five lake stations	Remote sensing long- term satellite data, LVBC and literature review	Hydro-climate analysis	**VI**

3.1.1. Bathymetry mapping: stage 1 (generate elevation model, DEM)

An accurate representation of the topography is essential for lake hydrodynamics and an elevation model contains elevation values at specific locations (Lophaven et. al., 2002). Conventionally, the values are collected from maps and field surveys. The DEM representation of the topography is then represented by a rectangular grid, also called an Elevation Matrix Structure, by a Triangulated Irregular Network (TIN), or by iso-depth contours (Shingare and Kale, 2013). NASA (STRM – Shuttle Radar Topography Mission) and U.S. Geological Survey (USGS) are open sources providing geological images, including satellite data with data available in different DEM file formats. In developing the lake bathymetry in Comsol we could not find such a source to generate a DEM. However, since Comsol can read a file of $m \; x \; n$ points (x_{ij}, y_{ij}, z_{ij}), in a regular grid where $i = 1,2, \dots, m,$ and $j = 1,2, \dots, n,$ the downloaded USGS data could be used to build the DEM file format and import it into CM and MATLAB (Paul et al., 2014).[7] The bathymetry mapping model also used digitized depth soundings, digitized depth contours, and modern echo-sounding data (Paul et al., 2019).

3.1.2. Bathymetry mapping: stage 2

Lake Victoria is, as mentioned previously, indeed a shallow water body (area 26,828 km^2). The ratio between its average depth ($H = 80\ m$) and its length ($L = 321{,}868.8\ km$ (200 $miles$)) is $H/L = 0.0025 \ll 0.05$, which is far below the standard shallowness parameter. For modelling of shallow lakes, bathymetry is also one of the key parameters (Paul et al., 2019) and here, expressing water depth, including wave motion, we assume the bathymetry $B(x,y)$ is a known contour map of the elevation of the “bed” or “floor” of the water body. We also assume free surface gravitational flow over a bottom topography fixed in time. The free surface vertical position from a reference datum, often referred to as celerity or wave height is then,

$$z = \zeta(x,y,t) \tag{1}$$

The water depth then becomes,

$$h(x,y,t) = B(x,y) + \zeta(x,y,t) \tag{2}$$

where z is the vertical coordinate, ζ is the celerity, and h the water depth, as presented in **Figure 4**. The additional map data needed includes coordinates of where in- and outflowing rivers meet the shoreline.

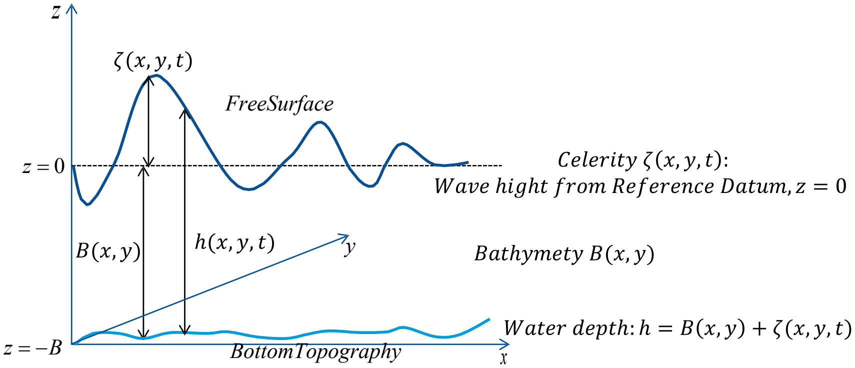

Figure 4: Vertical cut through the water body, where z is the vertical coordinate, ζ is the celerity, h is the water depth, B(x,y) is the lake bathymetry.3.1.3. Bathymetry mapping: stage 3 (set-up coordinate system)

The next step is to set-up the coordinate system. To do this we let θ be the latitude, 0 at the equator and +90 at the North Pole. φ is the longitude, 0 at Greenwich (UK) and increasing east (+). The lake

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[7] MATLAB is a high-level technical computing language and interactive environment for algorithm development, data visualization, data analysis, and numeric computation. KTH has a licence which was used for this thesis.

extent is small enough to be considered a plane normal to gravitational acceleration, x west-east, and y south-north. Then we have

$$(x, y) \approx R(cos\theta_0(\varphi - \varphi_0), \theta - \theta_0) \tag{3}$$

where R is the earth radius, 6371 km and $(\varphi_0, \theta_0 = 0)$ is some arbitrary central point in the lake.

3.1.4. Bathymetry mapping: stage 4 (bring in the measured data)

Lake water depth can be measured by several methods. Historically, soundings, i.e. measurement of depths, were carried out with hand-held lines. In the last few decades, the use of echo sounders has expanded enormously for its efficiency, accuracy and resolution for measuring shoreline and ocean and lake depths (Hell, 2011). Two different sources of lake depth raw data were available in this study: (i) Existing Admiral Bathymetry maps with iso-depth curves data**, Figure 5a**; and (ii) Modern digitized sets of points using echo sounders and satellite spectral remote sensing, which were obtained from the National Fisheries Resources Research Institute, based in Jinja, Uganda (NaFIRRI, 2014), **Figure 5b.** Additionally, to build the bathymetry, satellite data was used that covered the lake surface and extended 15 km east-west and south-north of the lake (**Figure 5b**).

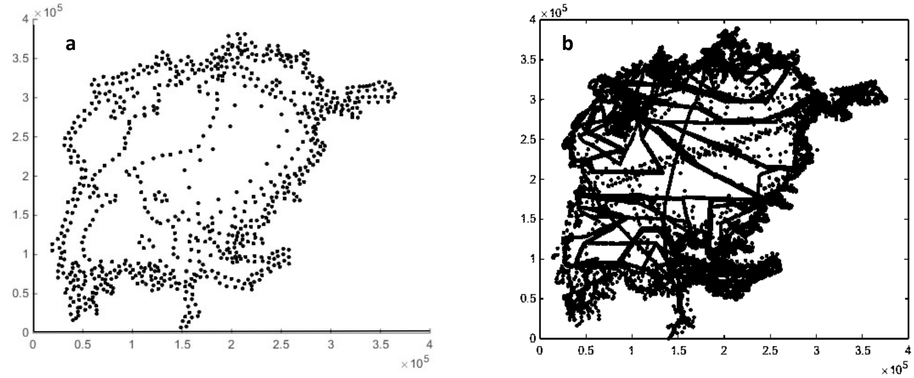

Figure 5: Different data sources to build the bathymetry of Lake Victoria: a) iso-depth curves (60 m depth); and b) NaFIRRI data.

The bottom slopes are the driving forces for the hydrodynamic model that consequently requires $(\partial B/\partial x\, , \partial B/\partial y)$ at the integration points of the finite element mesh. Since we do not have depth measurements for all integration points, this is a scattered data interpolation task that from the given depth data points will need to generate the finite element integration points that later will be used for the hydrological model. The task is accomplished by first creating an elevation matrix structure, a $N_x \times N_y$ rectangular grid of $B(x, y)$, dense and smooth enough to allow differentiation by differences. Since derivatives are needed, the task is challenging. Two methods were tried to create the table $B(X_i, Y_j)$, a Kriging method as implemented in the MATLAB DACE toolbox (in **Section 3.1.5**), and Delaunay triangulation of the points followed by interpolation and spatial filtering (in **Section 3.1.6**). The surfaces produced by Kriging showed small-scale wiggles in the slope calculation which could not be sufficiently reduced by choice of appropriate correlation lengths. The old Admiral iso-depth data, which had a well-defined shoreline $(B(x, y) = 0)$, must now be combined with the much more voluminous modern NaFIRRI data and there were obvious discrepancies in the shoreline locations, prompting the need to also generate a well-defined shoreline geometry using a bilinear transformation described in **Section 3.1.7**. All the steps of the calculation process are illustrated in **Figure 6**.

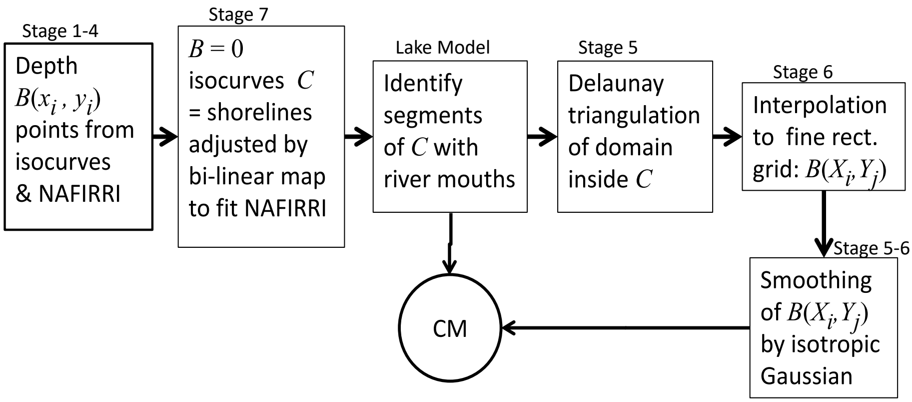

Figure 6: The research process on mapping the bathymetry.

3.1.5. Bathymetry mapping: stage 5

Kriging is a geostatistical interpolation technique, widely used in the design and analysis of computer experiments for response surface models, meta-models, or surrogate models. DACE is a MATLAB toolbox for implementation of a Kriging approximation (Lophaven et al., 2002). Kriging considers the data as samples of a stochastic field and sets out to obtain the best linear unbiased estimator to this field. It relies on the variogram $R(x, y)$ which describes how the covariance of the data at two points (x, y) and (u, v) depends on their relative location. DACE supports variograms of the form expressed in Equation (4),

$$R(x, y; u, v) = f\left(\frac{(x-u)^2}{R_x^2} + \frac{(y-v)^2}{R_y^2}\right) \quad (4)$$

where R_x and R_y represent the correlation lengths. These correlation lengths must be chosen to create the estimator, and the automatic procedure implemented in DACE is a nonlinear minimization. This approach becomes more difficult when the data distribution is very different from an essentially isotropic distribution. In our case, the points are on a few densely sampled curves so the distances between points on curves are much shorter than the distances between curves. Extensive manual trial-and-error choices of R_x and R_y could also not produce smooth accurate interpolants for lake topography from available data.

3.1.6. Bathymetry mapping: stage 6 (plot the bathymetry based on data)

The Delaunay triangulation is an essentially unique, minimum edge length triangulation (Lee and Schachter, 1980) of the convex hull of a given point set. The circumcircle of each triangle contains only the points which are vertices of that triangle. It is widely used for finite element mesh generation and for scattered data interpolation and can be computed in $O(n \log n)$ time for n points.

The Delaunay structure is used to interpolate a fine $N_x \times N_y$ rectangular grid covering the convex hull of the set of points. It was observed that some triangles in the shore regions had all vertices on the shoreline and thus vanishing depth. Near-shore points were then added manually to avoid such configurations. **Figure 7** illustrated the Delaunay triangulation on the iso-depth curve development data (**Figure 5a**). Some of the data points are shadowed in the plot by the depth-colouring.

The Delaunay Interpolant is only continuous (see **Figure 7**) but the model requires slopes, i.e. a smooth interpolant. Thus, the grid data were subsequently filtered by an isotropic Gaussian filter with a width chosen as a compromise between the distance between filtered and original data and smoothness.

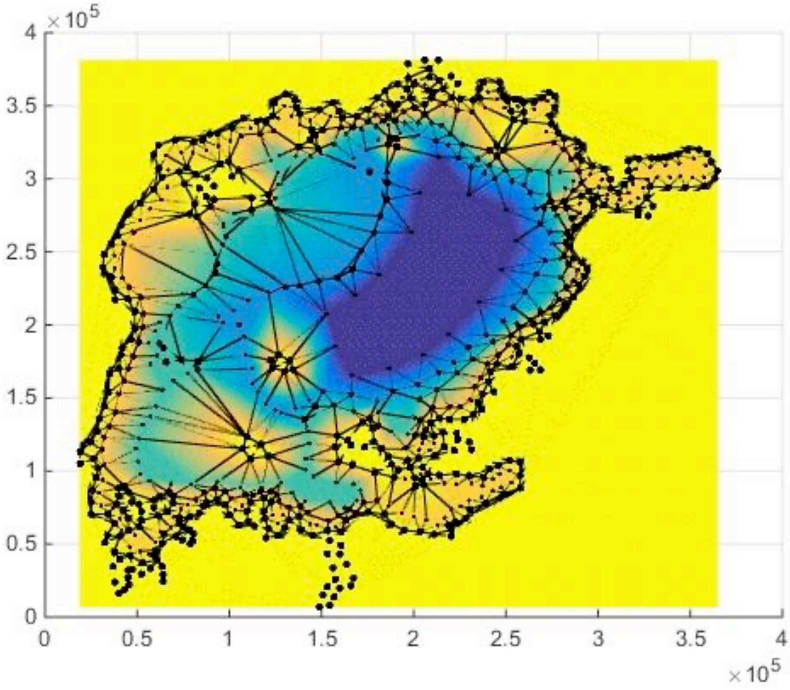

Figure 7: Delaunay triangulation and interpolation to iso-depth curve data.

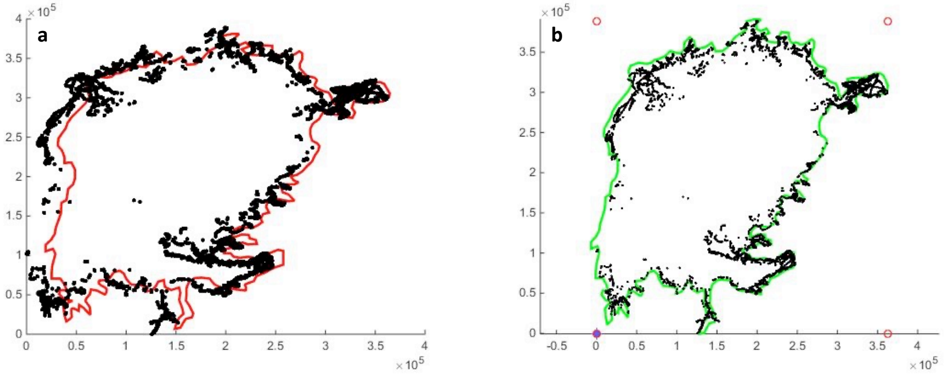

Figure 8: Mapped shoreline with new depth data (a) original, and (b) after bilinear transformation.

3.1.7. Bathymetry mapping: stage 7 (draw the shoreline based on data)

The shore geometry was well defined as a zero-depth level curve in the Admiral iso-depth data points. The NaFIRRI data points did not explicitly show the shoreline. When the iso-depth curve from the old data points (**Figure 8a**) were added to the set of NaFIRRI points with $0 - 10\ m$ depth, there was an obvious discrepancy in the shoreline, **Figure 8a**. To mitigate the problem, the locations of the new NaFIRRI data points were slightly modified by a bilinear transformation, defined by $2\ x\ 2$ control points, moved to obtain the best overall fit as judged by eye, **Figure 8b**.

The above development of the Lake Victoria bathymetry was made available on the Comsol website (Comsol, 2014; and see Paul et al. 2014). The development work further included extensive bathymetry raw data that included different systematic methods to develop a detailed bathymetry model for lake hydrodynamics more generally (Paul et al., 2019).[8]

3.2. Hydrodynamic model methods

3.2.1. Geospatial analysis

Geospatial analysis gathers values beyond mapping and displays geospatial information systems (GIS) data including satellite images (Chang, 2019). Spatial interpolation is used with estimated climate variables, i.e., rainfall, temperature, cloud, and drought to see the long-term climate effect in the lake and to study potential climate impact on potential lake stations (see **section 2.2**).

Bilinear interpolation was conducted to connect between lakeshore processing data points to help build the lake geometry in Comsol. Bilinear interpolation was based on the ratio of arithmetic averages where x_i and y_i represent significantly correlated variables:

$$f(x_i, y_i) = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} \tag{5}$$

Lake Victoria's rainfall over the lake experienced a predominantly positive trend over the 20th century. Precipitation significantly affects the lake region's water storage requiring a surface water hydrological model to analyze rainfall patterns (Akurut et al., 2014; Awange and Saleem, 2021). To understand how hydrodynamic processes and climate changes impact on Lake Victoria and its catchment areas are elaborated in **Paper VI.**

3.2.2. SWEs in conservation form for FVM

The Shallow Water Equations (SWEs) are a set of hyperbolic partial differential equations (PDEs) that describe fluid flow (Setiwoyaty and Sumardi, 2019) and can numerically be solved by finite volume method (FVM) and finite element method (FEM). The first investigation to build a hydrodynamic model for Lake Victoria was made on the high resolution schemes and by building a simulator based on the SWEs using the FVM. The 2D SWEs are often solved through FVMs in the presence of irregular lake shore topography (Cristo et al., 2021; Chowdhury 2008). The FEM simulator visualized the properly smooth lakeshore model and produced an initial model of Lake Victoria in Comsol. Different schemes and methods were tested to build the initial lake flow model in Comsol, which are detailed in **Paper III.**

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[8] Hamilton (2016) used similar old data and SRTM data sources to generate a DEM using raster interferometry and Kriging. In the fall of 2017, HARVARD Dataverse made the map accessible online (Hamilton, 2017).

3.2.3. St. Venant Shallow Water Equations (SWEs)

The SWE is based on the assumption of a shallow lake, i.e., $H/L << 1$, where H and L are the characteristic values for the vertical (i.e., depth) and horizontal length scales, respectively (Pedlosky, 1987; Paul and Rick, 2005). SWE describes motions in a thin layer of fluid of constant density in hydrostatic balance, bounded from below by the bottom topography (i.e., the bathymetry as developed above) and from above by a free surface. SWE needs a formulation that conserves mass and momentum if shocks or “hydraulic jumps” appear as in water in- and outflows or wind forced water movement (Hirsch, 2007). Any stable numerical scheme for solving hyperbolic equations must include dissipation, such as in upstream or discontinuous Galerkin schemes, or added as artificial dissipation, diffusion, or viscosity terms which are tunable by Hirsch (2007). The standard Galerkin formulation employed in CM requires such an explicit definition of the numerical dissipation.

The recipe is to choose the artificial kinematic viscosity v_A as isotropic and proportional to the maximal characteristic speed, i.e., a gravity wave speed plus advection with the element size Δ and an $O(1)$ tuning parameter μ, thus,

$$v_A = \mu \cdot \left(\sqrt{g \cdot h} + \sqrt{U^2 + V^2}\right) \cdot \Delta \tag{6}$$

The Coriolis acceleration factor γ follows from the earth’s rotation rate $\omega = 7.272 \times 10^{-5}\ s^{-1}$ with the geoid normal $\omega_\theta = 11.4 * 10^{-12}\gamma$ where $\gamma\ [m]$ is the distance to the equator (in meters). Given the above and keeping in mind the assumption of a shallow lake, the 2D SWEs can be obtained from the Navier-Stokes equations by depth integration and neglecting all variation in the vertical direction (i.e., neglecting diffusion of momentum due to viscosity and turbulence, etc.). This provides the depth-averaged two-dimensional flow equations detailed in **Paper I** that integrate momentum and continuity balance.

3.2.4. Lake level validation

The hydro-meteorological processes are summarized into the lake water balance (LWB) responsible for the changing lake water level (Melesse et al., 2008). The variability in river flows plays a significant role in hydro-meteorological modelling. Assuming that the shoreline L is fixed and integrating the SWEs over the lake area (A), we have:

$$|A|\frac{d\bar{\zeta}}{dt} + \int_L h\left(Un_x + Vn_y\right)ds = \int_A (P - E)dA \tag{7}$$

where the mean water level is calculated as:

$$\bar{\zeta}(t) = \frac{1}{|A|}\int_A \zeta(x, y, t)dA \tag{8}$$

The finite element model conserves a consistent approximation to the total water volume only if all equations are solved exactly. Exact solution is not possible for non-linear models and a check on the overall water balance is necessary to verify model simulations. The lake flow is determined by the initial and boundary conditions. The initial velocity field and celerity are arbitrarily set to zero at $t = 0$. The shoreline integral vanishes except at river inflows and outflows. The inflows and outflow are measured, probably quite reliably, so the uncertainty in their values should be much smaller than that in precipitation (P) and evaporation (E) values.

The data gives an estimate of the mean water level h, as shown in **Equation (9)** where the lake area is assumed constant in time. The CM simulation (**Equation 8**) was compared to the water balance equation:

$$A\frac{d\bar{\zeta}}{dt} = P - E + In - Out \tag{9}$$

P was extrapolated by LVBC and WRMA from meteorological stations on land but using methods unknown to us. E was modelled from temperature, wind, and geolocation also using data from land-based meteorological stations. The data were almost constant over 20 years (see **Figure 15**) which indicates that measurements were missing and/or were estimated from a 'typical' year. Smith and Semazzi (2014) and Kite (1982) also comment on the near constancy of lake evaporation values. The model equations and descriptions are detailed in **Paper II**.

3.2.5. Boundary Conditions for lake flow behaviour

The original SWE is a hyperbolic set of PDEs, so the number of boundary conditions to be prescribed for a mathematically well-posed boundary value problem is related to the characteristic velocities, see e.g. (Hirsch, 2007). The numerical finite element hydrodynamic model as described above includes an artificial kinematic viscosity ν_A. This means that the numerical model is parabolic and requires three boundary conditions, formulated in terms of the state variables and their normal derivatives. The model equations and descriptions are detailed in **Paper I**.

The shoreline is taken into account through a no-flow-through slip boundary. Viscosity ν_A is dominated by the gravity wave velocity and becomes small at the shore where the water depth is small. Since ν_A is small at the shore, a no-flux condition was used, which also annihilates the viscous contribution to mass flux. If mean velocities are large enough, no-slip conditions $U = V = 0$ should produce eddying wakes downstream of islands, however, they would dissipate quickly by artificial dissipation. The volume flux close to the shore becomes small except at inflows and outflows, so the difference in the numerical large-scale flow patterns is small between no-slip and no-flux conditions. The model equations and descriptions are detailed in **Paper I**.

In- and out-flow conditions is viewed as volume discharge modelled by a boundary volume flux condition for the celerity equation, i.e., a wall normal velocity and artificial viscosity contribution. Conversion of flow to flux or velocity requires an assumption of the width and the depth of the area of the flow cross section. This "width" of river mouth was chosen to be larger than the real width of the river mouth to avoid extremely small finite elements, yet small compared to shoreline geometry scales. The mass flux condition gives exact water volume conservation and no direct contribution to the momentum equations. The model equations and descriptions are detailed in **Paper I**.

3.2.6. Outflow conditions using weir (a low dam) and linear conditions

The inflow boundary conditions used, specify the amount of water entering the lake, unaffected by the lake water level. A "natural" outflow boundary condition, on the other hand, should react to the water level such that increasing levels increase the outflow. Data are available on both inflows and outflow and can be applied as boundary conditions. Since the outflow is regulated by a power station at Jinja, also other signals influence the actual outflow. Two candidates for "natural" outflow conditions are the weir condition (i.e., simulating a dam) and a linear outflow condition. For the weir condition models, outflow were modelled as if water was flowing over a weir or low dam of a given height (Zhao et al., 1994; Cozzolino et al., 2014). The flow increases non-linearly with water level and sets a definite steady-state water level, when inflow, evaporation, and precipitation is constant. For the linear condition models, the simplest choice is a linear relation to water level, measured from some reference height. Thus, we wish to find the initial water level $h(0)$ and the constant C in

$$Area \cdot dh/dt = In + Precip - Evap - Ch \tag{10}$$

$$h(0) = h0$$

such that the modelled outflow $Ch(t)$ matches the measured outflow $Out(t)$ as closely as possible. Both the weir and the linear condition models were fitted to historic data and both gave well posed steady state solutions as detailed in **Paper I**.

3.2.7. Transport of dilute solution

Advection is the process by which a tracer or contaminant moves with a fluid and dispersion is the combination of two processes, molecular diffusion, and turbulent mixing. Transport was modelled by the advection-dispersion equation, where a streamline artificial diffusion D_{art} was used to reach numerical stability:

$$\frac{\partial c}{\partial t} + U \cdot \frac{\partial c}{\partial x} + V \cdot \frac{\partial c}{\partial y} = -Q\frac{c}{h} + div\big((D + D_{art})\nabla c\big) \tag{11}$$

Here we have, vertically averaged velocity field (U, V), mean concentration $c(x, y, t)[mol/m^3]$, $D[m^2/s]$ is the dispersion coefficient modelling the turbulent mixing, D_{art} is a 2×2 tensor which adds dissipation in the streamline direction only (Hansbo, 1992), and $Q[m/s]$ is the net source of water volume, i.e., precipitation minus evaporation. At the shoreline, the flux vanishes, at outflow, the diffusive flux vanishes, and at inflow river mouths concentration is specified. The model equations and descriptions are detailed in **Paper II**.

3.3. Small object, wind hydrodynamics and transport

3.3.1. Moving small object in viscous fluids

To model the movement of organic pollution in the form of a small worm or microorganism, a model was built for the movement of small objects in water. Such locomotion of microscopic organisms in water (Lighthill, 1975; Berg, 1983; Childress, 1981) is an example of what hydrologists have called swimming at low Reynolds number which collapses Navier-Stokes equations to Stokes flow equations. To understand the motion of particles in Stokes flows, for instance bacteria swimming with the aid of flagella or cilia, or the movement of blood platelets modelled as spheroidal or ellipsoidal bodies (Seddon and Mullin, 2007), is important in the study of many biological systems. For this study, the aim was to build a fine-resolution model to understand the potential spread of pollutants and microorganisms in shallow lakes. The results correspond to two-dimensional numerical simulations of Stokes flow for a small swimming object at low Reynolds number. The dimension of the object chosen in this simulation was and elliptically shaped body of $0.002\ m$. This elongated body performs a wiggling motion modelled as a sinusoidal lateral (η) displacement of every point on its surface, which results in a traveling wave,

$$\delta\eta = v_0 \cdot \sin(k\xi - \omega t) \tag{12}$$

The wiggling amplitude v_0 is small, on the order of the worm's thickness, and the worm length is a few wavelengths $\lambda = 2\pi/K$. At these flows of low Reynolds number, the swimming motion of a small object depends on different wave parameters. The model equations, boundary conditions and explanations are detailed in **Paper V**.

3.3.2. Numerical tool for modelling data driven wind flows

The data-driven approach in this thesis to model how the wind patterns of Lake Victoria influence water flows, refers to analysis of wind speed and direction at a catchment scale by a combined use of numerical tools in CM. The objective was to extract, from decades of wind date, the typical seasonal and diurnal wind pattern, which in turn can be used to model how wind forcing influence the hydrology of Lake Victoria (**section 3.3.3**).

The data used were series of data recorded hourly at $03, 06, 09, 12, 15, 18, 21$, and 00 for 30 years from $1986 - 2016$ at the three Tanzanian stations. To avoid discarding significant amounts of data, synthetic data was added as follows: the cumulative distribution function (CDF) for each hourly reading was estimated from the existing data and missing data subsequently added as random numbers drawn from empirically based cumulative distribution function (CDF). The monthly averages of wind speed and directions are then available every three hours from 00 onward for N_y years. These data would constitute an $(N_y \times N_m) \times N_h$ data array $DS0$ (**Figure 9**) with $N_h = 8$ and $N_m = 12$. **Figure 9** shows three views of the data, with N_h daily readings for the $N_m = 12$ months and $N_y = 30$ years.

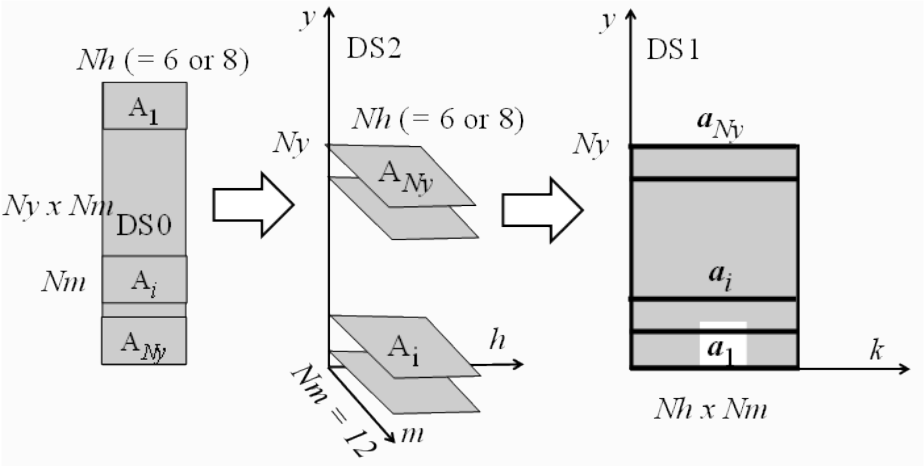

Figure 9: Numerical tool: The figure visualizes the organization of the monthly averages of wind speed and directions data under the dynamical array system.

The diurnal and seasonal variation is expected to produce strong components of $24\ hrs$ and 12 month period. Windspeed and direction were converted to north and east wind speed components u and v, since (u, v) vectors form a field admitting averaging. Three different data structures are considered for holding and approximating the data (u) (and correspondingly for v):

- a $(N_y \cdot N_m) \times N_h$ array $DS0$;

- a $N_y \times (N_m \cdot N_h)$ array $A\ DS1$;

- a set of $N_y \times (N_m \cdot N_h arrays\ A_i)$ DS2.

A row a_k of A corresponds to A_k by reshaping the vector into a $N_m \cdot N_h$ array:

$$A_i = reshape\ (a_i, N_m, N_h)$$

The u (east) and v (north) components of the wind are related to wind speed ws with angle θ measured clockwise from north:

$$u = ws \cdot sin(\theta), v = ws \cdot cos(\theta), ws = \sqrt{u^2 + v^2} \tag{13}$$

The aim was to extract the typical seasonal and diurnal wind pattern from this historical data. Suppose a fixed daily and monthly pattern is given by the unitary array $\boldsymbol{w}$. This amounts to finding the best rank-1 approximation in $L2$ norm of the whole $N_y \times (N_m \cdot N_h)$ data matrix A, readily computed by singular value decomposition SVD (Golub & Reinsch, 1970),

$$A = U\Sigma V^T, U = [\boldsymbol{u_1}\ \boldsymbol{u_2} \dots \boldsymbol{u_r}]; V = [\boldsymbol{v_1}\ \boldsymbol{v_2}\ \dots\ \boldsymbol{v_r}], \Sigma = diag\ (\boldsymbol{\sigma_1}\ \boldsymbol{\sigma_2} \dots . \boldsymbol{\sigma_r}) \tag{14}$$

where σ_i are the singular values of A, r its rank (= number of nonzero σ_i)

$$\boldsymbol{w} = v_1, \boldsymbol{w}^T \boldsymbol{a_i} = \sigma_1 u_{1i}$$

The approximation is a tensor product and the $\boldsymbol{u}$ and $\boldsymbol{w}$ are vectors of unit length, so we associate the units of the data, here m/s, with the singular value σ_1. It follows that only $\sigma_1 u_{1i} \boldsymbol{w}$ will have identifiable magnitudes. Also, there is a sign indeterminacy in $\boldsymbol{u}$ and $\boldsymbol{w}$: the result is the same when changing the signs of both. As such, we can move forward with an assessment of wind direction and speeds using a vector decomposition of the primary data. The detail descriptions and simulations are found in **Paper IV** (and see **Table 4**).

Table 4: Summary of the structure of the wind data designation under dynamical system array.

Systematic wind data	**Measurement stations**		
	Bukoba	Musoma	Mwanza
Number of years, N_y	1987-2016 $(N_y = 30)$	1987-2016 $(N_y = 30)$	1987-2016 $(N_y = 30)$
Number of months, N_m	12	12	12
Number of every three-hour dataset, N_h	8	8	8

3.3.3. Wind induced hydrodynamics

Wind-driven flows are important in shallow lakes (Chen et al., 2020). The wind speed increases with height in the atmospheric boundary layer, created by friction against the surface. The transmission of momentum between wind and water is generally formulated as shear stress, depending on the wind velocity $\underline{v}$ relative to the water velocity, the density of air, and a drag coefficient (Nelen & Schuurmans, 2020; Overmeen et al., 2021). The standard wind measurement height is at 10 m above the lake level (Homoródi et al., 2012), the drag coefficient C_d is indicative of the mean wind speed vertical gradient and increases with water surface roughness. The surface stress vector τ becomes

$$\tau = \rho_{air} C_d(|W_{10}|)|v|\underline{v} \tag{15}$$

The model is only valid for small wind induced waves, i.e. $|W_{10}| < 12\,m/s$.

The shallow water equations (SWEs) can describe the depth-averaged two-dimensional flow (Paul et al., 2019). The wind force (F_E, F_N) added to the SWE model that was developed in **Paper I** (Paul et al., 2019) is expressed by:

$$\boldsymbol{\tau} = (F_E, F_N) = \rho_{air} \cdot C_d \cdot \sqrt{v_E^2 + v_N^2} \cdot (v_E, v_N) \tag{16}$$

The wind stress model parameters are given in **Table 5** and a detailed model description and results from the simulations are provided in **Paper III**.

Table 5: Wind stress model parameters.

Parameters	Unit	Value	Description
τ	N/m^2		Surface shear stress
ρ_{air}	Kg/m^3	1.225	Air density
$\underline{v} = (v_E, v_N)$	m/s	$(W_{10}E - U, W_{10}N - V)$	Lake surface velocity (U, V) W10 10m above the mean water surface
C_d	$-$	$(0.8 + 0.065 \cdot W_{10}) \cdot 10^{-3}$	Friction coefficient
W_{10}	m/s	$\sqrt{(W_{10}E)^2 + (W_{10}N)^2}$	Wind vector $(W_{10}E, W_{10}N)$

3.3.4. Transport process: Step-by-step numerical procedure

In 1905, Swedish oceanographer Ekman (1905) developed a theory of balancing wind stress, viscous stresses, surface elevation, and the Coriolis effect. Note that the horizontal Coriolis effect is essentially absent in Lake Victoria as it lies on the equator, which means that there is no Ekman layer to account for. Fredholm, another well-known Swedish mathematician, performed a more accurate analytical approximation, which was quoted by Welander in 1957 (Welander, 1957), who extended the Ekman model and indicated a numerical step-by-step procedure for prediction of wind-driven flow, which was used here.

Consider the transport of a tracer element, for instance pollution particles, with concentration $c(x, z, t)$ moving in a wind-driven shallow water flow. The notation here suppresses the variation with y. The flow is modelled by the hydrostatic equations (Ekman, 1905),

$$u_t + uu_x + g\zeta_x = v_{zz}u_{zz} \tag{17}$$

$$u(0) = 0, \nu u_z(z = h) = \tau \qquad (18)$$

$$u_x + w_z = 0 \qquad (19)$$

$$\zeta_t + \int_0^h u_x dx = 0 \qquad (20)$$

where $h(x)$ is the depth, $\zeta(x,t)$ the celerity, g gravitational acceleration, v_{zz} vertical turbulent kinematic viscosity, and τ surface stress from wind divided by water density. The vertical velocity w is recovered when needed, from the continuity equation. The transport of the tracer is modelled by an advection-dispersion equation,

$$c_t + uc_x + wc_x = D_{zz}c_{zz} \qquad (21)$$

For shallow lakes, it can be assumed that the vertical turbulent dispersion coefficient D_{zz} is so large that the concentration of pollutants will be close to constant in the vertical direction. The time for diffusion to average $c(x,z,t)$ over the depth h is $t_{eq} = \frac{h^2}{D_{zz}}$. In wind-driven flows, by the shallowness, it can be assumed that the u-profile is also in quasi-equilibrium, so turbulent stresses balance the hydrostatic pressure gradient. The two quasi-equilibria are consistent when the turbulent Prandtl-number is around 1 which is commonly assumed.

The variation of u with z creates horizontal dispersion. For time-scales larger than t_{eq}, a discrete model, Eq. (21) is proposed, based on a time-splitting of the two transport processes: horizontal advection and vertical dispersion, where $< \cdots >$ is shorthand for averaging over z, $< \bar{f}(x) > = \frac{1}{h}\int_0^h f(x,z)dz$

$$\bar{c}(x, t + \Delta t) =< \bar{c}(x, t - u(z)\Delta t) > \qquad (22)$$

Eq. (22) is in turn approximated by the 1-D advection-dispersion model

$$\bar{c}_t + \bar{u}\bar{c}_x = D_h\bar{c}_{xx} \qquad (23)$$

$$\text{where } D_h = \frac{h^2}{D_{zz}} < (u - \bar{u})^2 >$$

The derivation of Eq. (23) from Eq. (22) follows from Fourier expansion in x, n steps of Eq. (21), expansion in powers of Δt, and interpreting the result as if $n\Delta t \to 0$ although n is large and Δt is constant. The more detailed model description and simulations are in **paper IV.**

3.3.5. Transport process: hydrostatic stability

The standard shallow water equations follow from replacing u by $\bar{u}$:

$$\bar{u}_t + \bar{u}\bar{u}_x + g\zeta_x = v_{xx}\bar{u}_{xx} \qquad (24)$$

$$\zeta_t + (h\bar{u})_x = 0 \tag{25}$$

They are
well-posed. Standard linear finite elements produce discrete equations much like the central differences on first-order derivatives that are well known for being prone to wiggles. Finite difference schemes use staggered grids or other tuned difference schemes to avoid that. Eq. (25) perform well in CM as long as v_{xx} is chosen to exceed the amount of dissipation necessary to avoid the wiggles,

$$C\left(\sqrt{gh} + max(\bar{u})\right)\Delta x \tag{26}$$

where C is a constant depending on the finite elements chosen, Δx is the element size, and $\sqrt{gh} + max(\bar{u})$ is the maximal characteristic velocity with $\sqrt{gh}$ the speed of isotropic wave propagation. Comsol implementation of the continuity equation was done by modifying the continuity equation

$$u_x + w_z = 0 \text{ into } \tau w_t + u_x + w_z = \mu w_{zz},$$

Setting $\tau = 0.01\ sec., C = 0.1$ gave acceptable results. The detail descriptions and simulations are in **Paper IV**.

4. Results

4.1. Lake Victoria bathymetry in Matlab and Comsol

The model bathymetry was compiled from several sources and merged as describe in **Section 3.1**. The final geometry is based on the digitized iso-depth curves, locations of islands, and the NaFIRRI echo sounding and spectral satellite data. It is shown in **Figure 10** as a shaded lake bathymetry of the whole lake with lake depth (see also **Figure 5**). The governing equation under PDE solver in CM adopted the SWEs method for the $2D$ Lake Victoria model. **Figure 10** shows that the lake's bathymetry is steeper in the east than in the west. **Figure 10** lake bathymetry steps down from north to south toward the deepest point (80 m) in the basin. The two topographical models were used to perform water flow analysis of Lake Victoria, with the two main discharges being the Kagera river in the west, which is the main river inflow to Lake Victoria, and the other being the Nile in the north, which is the only outflow of Lake Victoria (**Figure 3b**).

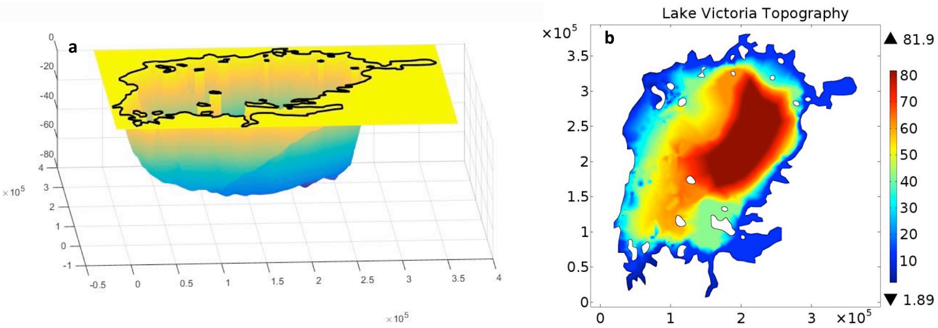

Figure 10: Lake Victoria water depths based on iso-depth curves and NaFIRRI bathymetry data: a) Matlab model; b) Comsol model.

4.2. Initial lake model in Comsol

The adopted method with initial boundary condition is appropriate in Comsol for a rain-drop model when the rain-drop starts diffusing from the very beginning (at $t = 0s$) as depicted in **Figure 11a**. The flow velocity of the lake water surface is increasing with respect to time, as well as lake water level increasing with different lake depths (**Figure 11b**). There are no river inflows and outflow boundary conditions applied except initial values imposed and no lake flow discharge from the lake basin (**Figure 11**). Therefore, the flow velocity developed across the lake and after a certain time period, the lake water flow becomes placid and returns through a dissipative mode to the initial stage.

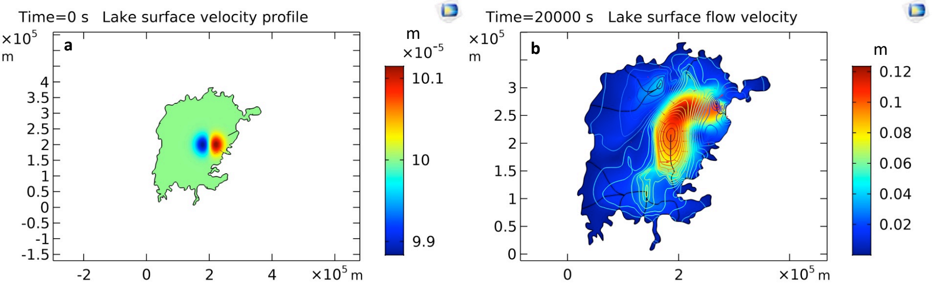

Figure 11: Lake Victoria initial rain-drop model in Comsol, a) initial time at t= 0 s, b) Dissipative final time,t = 20000 s.

4.3. Hydrodynamics flow behaviour

The full lake regional flow model analysed over the long run using mass conservation and momentum equations and integrated with respect to depth (**Figure 10b**). In the initial stage, flow analysis was defined by initial values under initial boundary conditions and boundary values defined by river discharges under Dirichlet Boundary Conditions (DBC).

Initial values of three dependent variables (velocity, U and V; reference water level, zeta) was initially adopted to serve as an initial guess of water flow (**Figure 12**) alongside the DBC adopted river discharges, which prescribe the lake flows for the whole lake area (**Figure 12a**). The simulations shown in **Figure 12** used Neumann conditions to guarantee conservation of water volume.

Figure 12 shows that both data and boundary conditions are adequate and water flows across the lake without encountering a barrier. There are numerous shallow water simulators around, but for many of them the time step is limited by numerical stability properties. Here, the simulation used monthly averages for 51 years of data from LVBC and WRMA on precipitation, evaporation, inflows and outflows and the situation in 2000 was presented in **Figure 12** (outflow boundary condition and steady-state analysis are discussed in **section 4.4**). The lake streamline flows in **Figure 12a** shows a smooth flow from the tributaries (river inflows) to the outflow with no barrier. It is indeed close to a flow with vanishing vorticity, $\omega = U_y - V_x$, as substantiated by the vorticity plots, because the artificial viscosity with flux boundary conditions cannot produce vorticity. The only sources of vorticity are wind stress, bottom friction, and the weak singularities at the boundary points between inflow and no-flux shore portions. In **Figure 12b,** vorticity shows up as small red and blue regions, i.e. positive vorticity on one side of the tributary, and negative on the other.

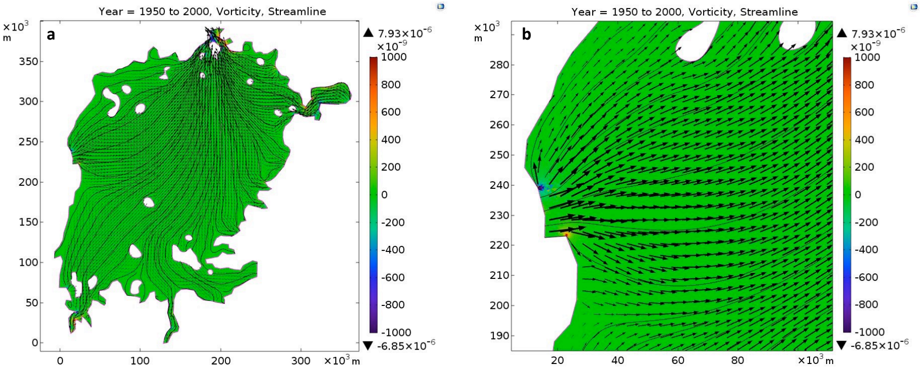

Figure 12: Flow patterns of Lake Victoria: a) Lake flow with streamlines; and b) zoom on Kagera river mouth region.

4.4. Water level validation

Generally, the lake water level is influenced by the seasonal and climate variation, lake hydrology, and land uses. It has changed significantly over the years and rose by about 2.8 m in 1962 – 1964 (**Figure 13**). The data provided by LVBC and WRMA contains discharge from the tributaries (In), the outflow at Jinja (Out), the precipitation (P) and Evaporation (E) for the 57-years period 1950 – 2006. The data demonstrated an estimate of the mean water level h, as shown in **Equation (10)**. To avoid excessively short time steps, the daily data were low-pass filtered by a monthly moving average.

In **Paper I**, the optimal values are found by minimizing the sum of squared monthly discrepancies. The agreement between $Out(t)$ and $Ch(t)$, as shown in **Figure 14**, is excellent in view of the simplicity of the model.

4.5. Lake water balance

Large variations in rainfall are, as was shown in the previous section (**Section 4.4**), responsible for changing the lake water balance and water flow movements. Historical data on annual and seasonal stream flow variations and particular monthly fluctuations for specific river discharges were analyzed. The differences in input and output result in differences in lake water level. Since the sum of the terms (P-E+In-Out) is much smaller than the individual terms, even a small bias in one of these terms could lead to large variations in the water balance. The components of the water balance and the measured water level (WL) from 1950-2000 are shown in **Figure 15**. As can be seen from the diagram, precipitation, and evaporation are larger than lake inflow and outflow. As expected, there is strong correlation between annual precipitation and total river inflow, and between lake outflow and lake water level, detailed in **Paper II**.

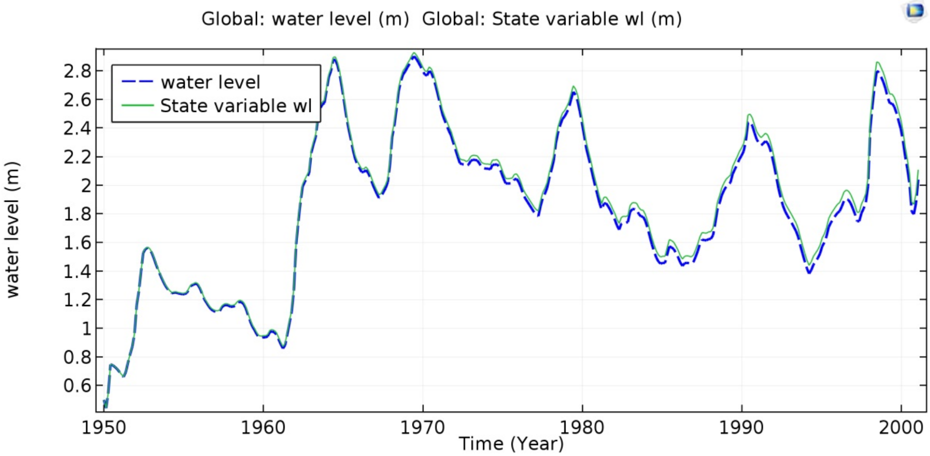

Figure 13: CM mean water level (wl) and water balance Eq. 10 (water level).

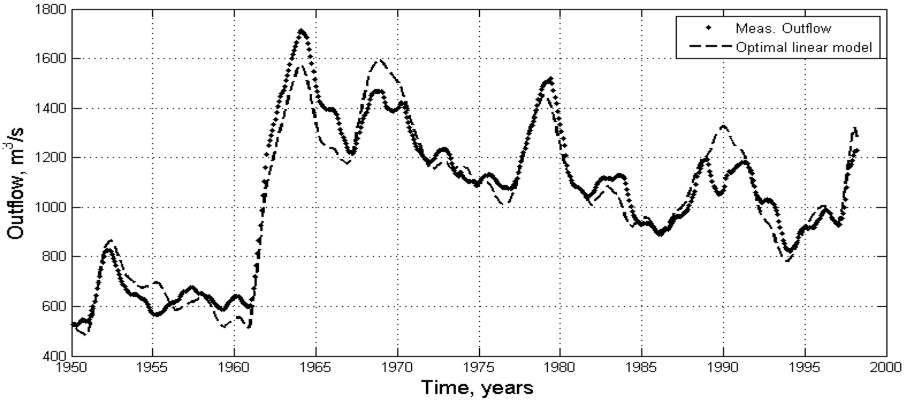

Figure 14: Mean water level simulations comparing fitted linear outflow model to measured outflow.

4.6. Tracer transport model

The advection-dispersion model (**Figures 16a-f)** shows the situation in the four rivers in 1950-2006, 1960, 1990, and 2006. Simulating across fifty-seven years (1950-2006; **Figure 16a**), the model shows that transport along the Kenya and Uganda lakeshore is much faster than in Tanzania. Eleven years of tracer transport (1950-60; see **Figure 16b**) did not affect lake zone areas in Tanzania but Kenyan lake areas were seriously affected. Water from the Kenyan rivers travels much faster, due to the shallowness along the Kenyan shore (**Figure 16a-e**), which, as we should note, has many factories and industries producing pollution. The analysis also shows that Kagera water takes 50 years to reach Jinja, traveling along the Ugandan shore and then entering the center of the lake (**Figure 16a, d, e, f)**. This slow movement strongly influences solute transport, which causes the lake to have high concentrations of pollutants from different sources to affect the Ugandan part of the lake's water.

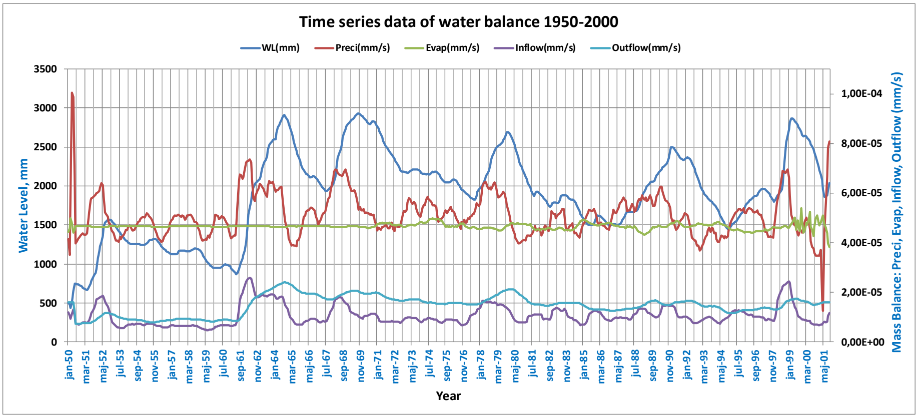

Figure 15: Lake Victoria water balance by measured data: water level, monthly inflow, outflow, rainfall, and evaporation for the period 1950-2000 (WRMA and LVBC).

Figure 16e visualizes only the solute transport from the rain from the wet period $(1950 - 2006)$ and with rain water volume as initial concentration with the bulk water simply dispersed outwards from its source in a diffusive manner. The dispersion is larger in the wet period than other periods, so, the lake water circulation are importantly affected by seasonal events (**Figure 16d, e**). To clarify the effect in the wet period, we have added extra diffusion in the model $(0.025\, m^2/s)$ for the Kagera river flow (**Figure 16f**), and the results show the solute diffusion interacting with the solvent in water.

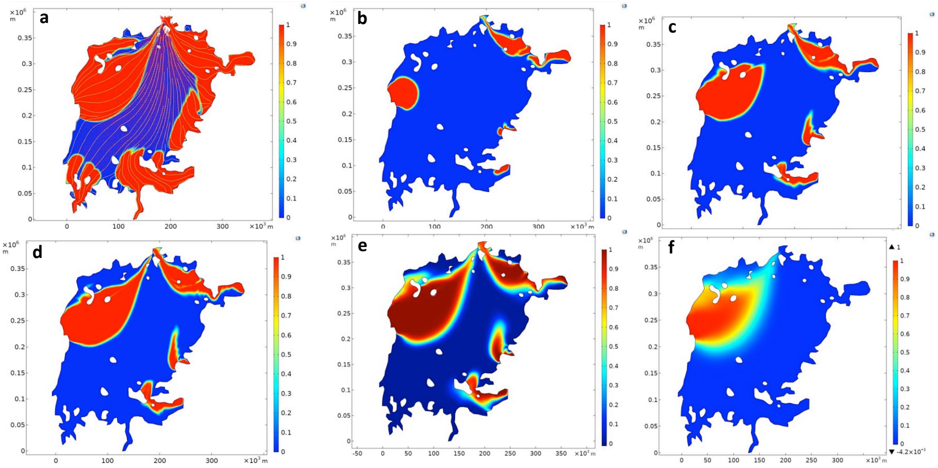

Figure 16: Tracer concentration on Lake Victoria from 1950 to 2006: a) all inflows with all seasons; b-d) four inflow rivers and all seasons; e) solute diluted by rain in wet period for four major tributaries; f) added extra diffusion in Kagera river water.

4.7. Modelling a microorganism moving in a fluid

We have computationally visualized the flow of an object performing a sinusoidal wave with body deformation. This models a wiggling worm or other microorganism and can thus be used to model the spread of organic pollution in water and fluids. The geometry of deformations of the moving object depicts the velocity profile at different time steps (**Figure 17a-b**).

The two different images show the small object's flow velocity with the moving mesh (Figure 17a-b). The deformation of the object depends upon its boundary conditions. Using low Reynolds number approximations, force and torque are applied on the swimming object from the global coordinate system and we get the linear and angular velocity by translating changes in velocities and motion in the local coordinate system back to the global coordinate system. The velocity direction changed periodically with the object motion, which produced a movement in the direction of the object's wave motion. The $x - direction$ displacement refers to the moving object displacement per time period. It shows that the deviation of the motion from an average straight line is sinusoidal in nature.

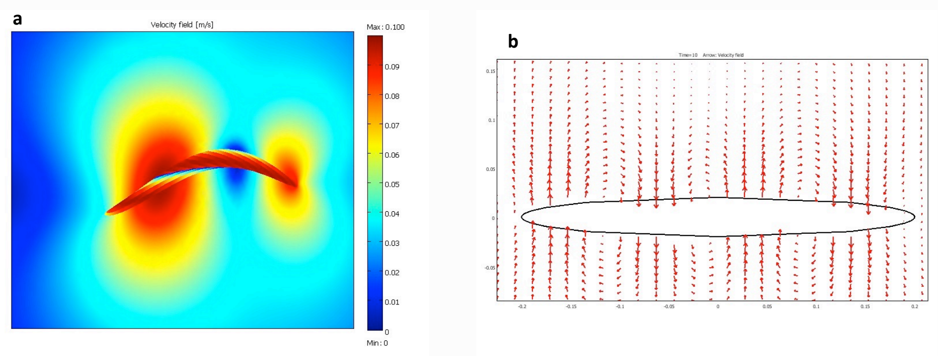

Figure 17: Velocity profile for moving object, a) particle moving in a fluid, b) velocity changing by particle movement.

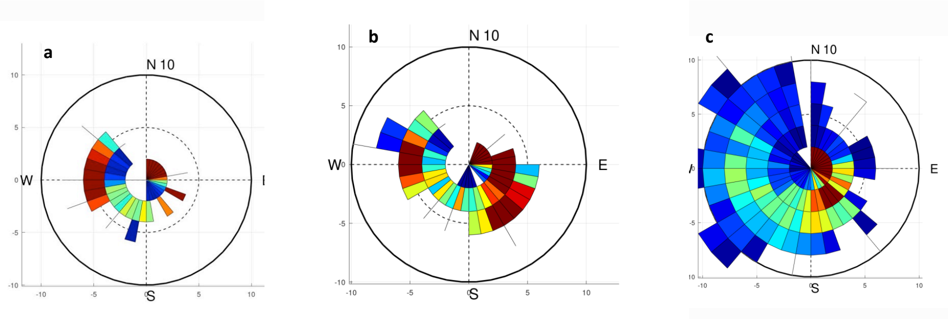

Figure 18: Wind roses at the three Tanzanian weather stations based on data from January 1986 to December 2016. Wind speed 10 m⁄s is marked by a circle, a) Musoma, b) Mwanza, c) Bukoba.

4.8. Wind pattern assessment

The three Tanzanian weather stations with data recorded for 30 years from 1986 – 2016 was used to analyse wind patterns (**Figure 18**). The wind rose, which is a diagramme to analyse and visualize prevailing wind directions and speeds. The absolute frequency in a $speed\ \times direction$ interval is color-coded, red being the highest, with no color for zero occurrences. For Musoma, the dominant wind direction is northwest and south (**Figure 18a**). In Mwanza, the northwest (NW) and east winds were more dominant than in other directions (**Figure 18b**). The Bukoba wind was strong in all directions (**Figure 18c**). The vertical wind shear of Mwanza was much stronger than at Musoma and blows through SE and NW, which was in agreement with the observation that the dominant wind in the lake is SE and NW.

4.9. Wind induced hydrodynamic flow modelling

4.9.1. Wind model simulations

The wind-driven flows for the wind velocities assessed from the weather station data could potentially be much larger than the water inflows and outflows. We utilized a modelling framework to explore the possibility. For long-time steady wind forcing, the water level gradient approaches the W_{10}-wind direction, and the water level difference across the lake is proportional to W_{10}^2 as seen by the expressions in **Table 5**.

Figure 19a shows the effect of $3\,m/s$ SW wind, "turned on" and kept constant thereafter. **Figure 19b** shows how this creates a *seiche*: a damped sloshing of the whole lake with a period of about 6 hours. This is consistent with the estimate of $3.5\ hours$ for the time it takes for a gravity wave to cross the lake. The artificial viscosity and the bottom friction are the dissipative processes responsible for the damped surface motion. Thus, a flow velocity driven by transient in- and outflows with steady wind makes little effect on lake water level. It follows that temporally and spatially resolved wind field data are required to obtain the proper influence of wind on lake flow patterns.

4.9.2. Murchison Bay wind driven flow

The wind-driven flow was used to calculate the wind current velocity for the flow field in MB. The model equations have been added to the time-dependent momentum equations and the transport of Diluted Species developed in Paul et al., (2019) and Paul & Oppelstrup (2020). Basically, when Nakivubo channel flow enters, the mixing can be set in motion by the wind flow fields that is measured at the Entebbe station. The Kagera River flow is exceptional in the dominance of open lake processes affecting the behaviour and mixing of its plumes as it enters into the lake directly **(Figure 20)**. The Kagera, Entebbe, and Nakivubo channels have been used as the source of wind flux, transportation, and flow species reactions in the domain, and added boundary to the deeper lake zone and to the outer boundary. Finally, a moving mesh was used for the time-dependent deformation allowing the geometry to change its shape due to the dynamic nature of the problem (**Figure 20a-b**).

The simulations showed the concentration with the wind-induced flow has little influence on the integrated flow model. This was demonstrated using a "typical" wind and studying the reaction of a diluted species tracer transportation signal from the Nakivubo channel. The simulations also showed the small velocities of the wind-induced flow in the integrated flow model. The tracer model was run with velocities up to one thousand times larger to make the tracer movement visible. The model shows (**Figure 20c-d)** the complex flow mixing between wind direction and resulting flow caused by the inherent flow dynamics and the depth variation of the region.

4.9.3. Pollution moved by hydrostatic model

The vertically integrated wind driven flow model moves pollution. The velocity profile and surface slope equilibrate quickly as sketched in **Section 3.3.4** and in **Section 3.3.5**. The numerical model shows the actual velocity profile through the water column and how it leads to horizontal dispersion. Simulations with the depth resolved using the hydrostatic flow model with free surface have been performed for a vertical cut through a gently sloping bay. The originally still water is set in motion by a constant wind $5\,m/s$. A passive tracer, originally with horizontal Gaussian profile, is transported, with very small dispersion (**Figure 21a**) to a very large vertical dispersion (**Figure 21 b-d**), the process approaches pure horizontal diffusion.

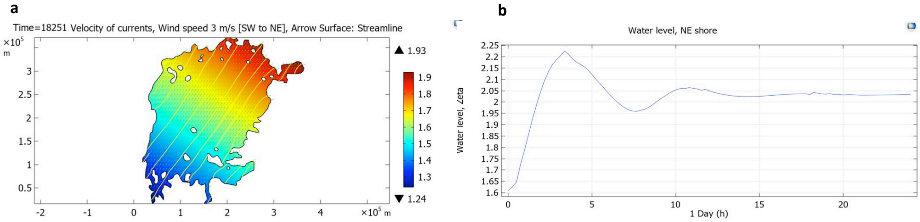

Figure 19: One-day constant wind speed simulation, SW wind force 3 m⁄s, a) water level after 5 hrs, b) one day water level variation at NE shore.

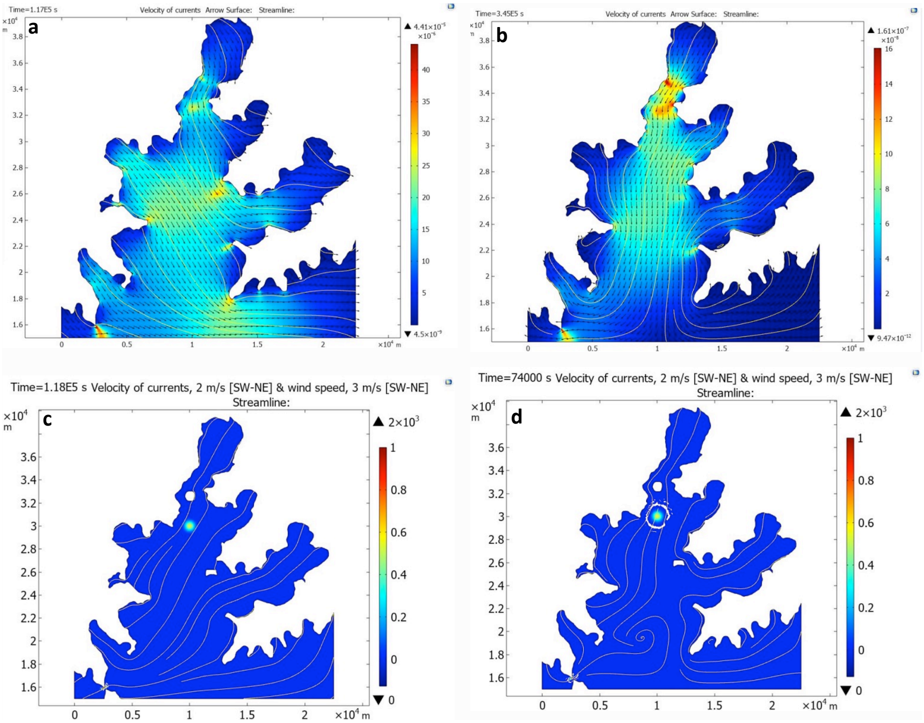

Figure 20: Murchison Bay wind-induced flow from the Entebbe station data, and movement of a tracer blob. At different times a) wind-induced pollution movements, b) wind-induced pollution movements, c) imposed tracer blob, d) imposed tracer blob.

Figure 21a shows the tracer transported by advection alone. If the depth were constant, the plume would follow a constant parabolic profile, as seen in **Figure 21b – d**, with vanishing net flow. The blob moves with the bulk velocity and additional dispersion. The diluted species mixes from top to bottom at the same advection speed as the bulk water. Finally, we note that there is a necessity of stabilization of hydrostatic depth resolved model in Comsol and this was achieved through artificial dissipation. Based on the wind patterns and our extreme wind events results it appears that the depth-resolved hydrostatic stability models could account for transport processes in the lake's hydrodynamics.

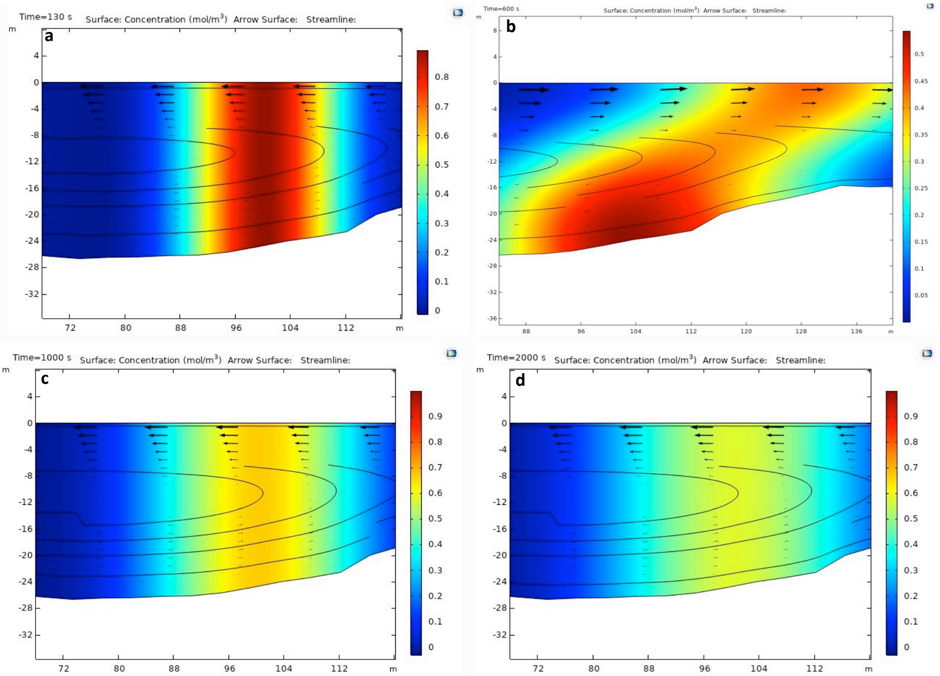

Figure 21: Hydrostatic flow model; a) pollution blob dispersion with small vertical dispersion at t = 130 sec; b) to d) Concentration with large vertical dispersion, at t= 600,1000,and 2000 sec.

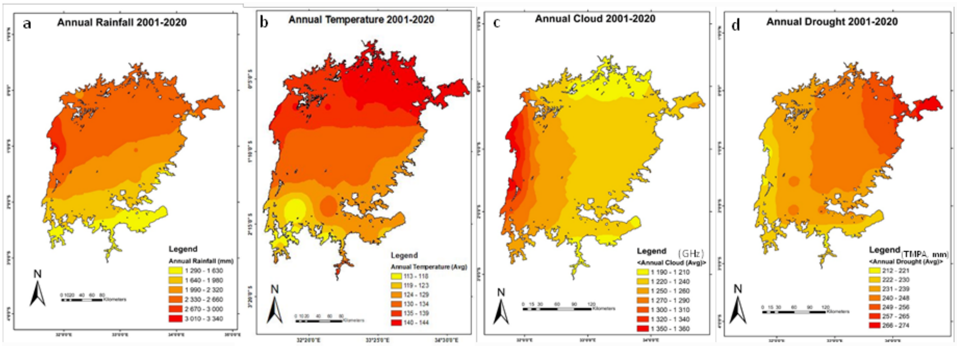

Figure 22: High-resolution meteorological-climate 21 years datasets visualized climatologic patterns of Lake Victoria, a) rainfall, b) temperature, c) cloud, and d) drought.

4.10. Climate scenarios

Using the model to analyse four major climatologic components of Lake Victoria—rainfall, temperature, cloud, and drought—a complex hydro-climatologic distribution is manifested (**Figure 22**). A more intense temperature gradient concentrated over the north, half-east, and half-western part of the lake and over the south-western clod extending into the middle half of the bottom lake. Similarly, an extensive precipitation gradient concentrated over the middle of the west and the middle of the upper lake while less precipitation appears in the south and western portions of the lake. However, an intensive drought appears where the temperature gradient is higher, and an intensive cloud is projected where extensive precipitation has appeared. Where temperature is higher the evapotranspiration is visibly higher, which makes drought appear more visible with drought effects higher in the summertime whereas cloud effects are higher in winter. The shallower lake region heats up much faster than the rest of the lake during the daytime, so water is likely evaporated faster producing clouds for precipitation. Lake effects on cloud moisture thus paralleled the indicated precipitation and temperature. What is not available for hydrologic research is detailed information on lake effects by seasonal precipitation, temperature, winds, clouds, vapor pressure, etc., which needs to be combined to improve climate change projection scenarios.

5. Discussion

This dissertation has aimed to contribute new insights into lake hydrodynamic processes and pollution transport in shallow lakes through using Lake Victoria as an intriguing and important case study. Chapter 3 provided an overview of the mathematical methods that were used, how empirical data was integrated and used, and Chapter 4 provided the key results from the six papers that forms the base of this thesis. Taken together the thesis has shown how mathematical models of metaphysical systems such as the shallow lake of Lake Victoria can be modelled to take into account complex lake bathymetries and shorelines, how long empirical data series of lake flow, water levels and hydro-climatological processes can be integrated and how the transport and dispersion of pollutants and nutrient particles can be modelled using numerical methods and hydrodynamic equations to model the behaviour of shallow lakes over time.

Here three main findings will be discussed from a more general perspective of providing advice in developing and using hydrological models for shallow lakes. Firstly, how to validate shallow water lake models and also the challenges and opportunities that arise in taking into account multiple lake flow data sources. Second, how river discharge is crucial in modelling pollutants and tracer dispersal, but also how wind is a rather complicated factor to account for. Third, and from a more integrative perspective, how the numerical hydrological models that sits at the heart of this thesis has opened the possibility to build more integrative models to understand how shallow lakes like Lake Victoria is influenced by increased environmental variation at the catchment scale due to climate change.

5.1. Validating models and handling various lake flow data sources

The lake bathymetry development model developed in Chapter 3 and in Paper I-III used a vertically integrated SWE model. The simulation considered over fifty years of daily measurements. The interest here is to discuss how monthly averages of inflow and outflow, precipitation, and evaporation were used to validate the model. While this is a not statistical test, the model was shown to faithfully reproduce monthly averages, indicating its validity. The procedure to reach this conclusion is important to highlight as it has general value for the development of shallow water lake models.

Data on both inflows and outflows are available and can be applied as boundary conditions. It could be argued that the outflow should react to the lake water level such that increasing levels increase the outflow. However, the Jinja power station—controlled by Ugandan state agencies—regulates the lake outflow, which means that other signals than natural ones influence the lake's actual outflow, a not uncommon situation in practical modelling. To solve this, two models were tested, a weir boundary condition and a "natural" outflow condition (as detailed in **Paper I**). The former expressed that the flow over the weir, a wall from the bottom of the river blocking its flow, increased non-linearly and data could be fitted to generate a bounded problem with good agreement. The linear outflow condition—i.e., a “natural” outflow condition without the Jinja power station—was fitted to historic data which mathematically provided a well-posed steady-state solution (as shown in **section 3.2.6**). In view of the simplicity of the “natural” model (**Figure 14**), the agreement between actual outflow and what the model could re-produce using measurement was excellent (detailed in **Paper I**) and serves as a method for other shallow lake modellers.

Lake water balance is however also affected by hydro-meteorological processes, which complicates modelling but also opens towards using meteorological data series that can span decades and help develop more accurate models. In our case of Lake Victoria, we had access to the WRMA and LVBC historical data (1950 – 2000), but as shown in **Figure 15**, they differed in water level with about 8 % (Kayombo and Jorgensen, 2005). The largest difference appearing at the Jinja outflow, the only outflow and where the power station is located. We also note in **Figure 15** that direct precipitation and evaporation dominates the dynamics of the lake water balance. How to handle this in practical modelling? A first step is to recognize scale of uncertainty. While there are uncertainties in river in- and outflow data, these are most probably dwarfed by the uncertainties in precipitation and, above all, evaporation, which are notoriously difficult to measure above a lake and the ocean. Thus, even if precipitation and evaporation comes from reliable sources, the raw data still needs significant manipulation before being used for modelling (as also described in Sewagudde, 2009). On one hand, data from actual geographical measurement positions has to be extrapolated to cover the whole lake area in order to be used in the model (explained in Chapter 3.1 and Chapter 3.2 above; details in **Paper I**-**II**). Furthermore, data series that measure the same things can differ. For Lake Victoria, it was clear that the WRMA, LVBC and Sewagudde data series differed significantly in terms of outflow, resulting from differences in evaporation estimates, despite referring to almost the same period and using data from the same source, the Lake Victoria Environment Management Project (LVEMP). For this problem, the thesis developed a protocol of comparing water balance data to capture long term trends (**Paper II**). Comparing 1956/57 to 1977/78 (Gibb & Partners, 1984), it was possible to estimate that the annual rainfall directly into the lake was increasing. This made it possible to show that rainfall patterns and water levels changed over the period from 1950 to 1980 (Kite, 1981; Kite, 1982; Sene & Plinston, 1994) and in the period 1995 to 2007 (Vanderkelen et al., 2018) with mean annual rainfall increasing by 33 % and lake water level rose by around 2.8 *m* between 1960 to 1964 (Kite, 1982; Sene & Plinston, 1994).

Apart from capturing trends over the longer term, the thesis also used a hydro-meteorological survey in **Paper II** to make detailed analysis of all the tributaries and precipitation over the lake. Important to remember here is that river discharge is often a crucial signal (rather than simply precipitation) for climate change analysis because changes in rainfall are usually amplified in runoff (Awange et al., 2008). For the Lake Victoria, the thesis analysed data from the four most influential rivers Kagera, Mara, Noiza and Gucha-Migori or Yala of Lake Victoria that contributes 33 %, 5 %, 15 %, and 5 %, respectively (FDMT, 2018). With a total of 58 % of all inflows, these four major rivers showed high seasonal fluctuation in the years 1952, 1963, 1968, 1988, 1990, and 1998 and that daily seasonal variability is higher in these rivers than in other rivers (**Figure 23**). The mean annual seasonal flows of the Kagera and Gucha-Migori rivers are much higher than those of the other rivers, especially in the dry period, at which time the contribution from all rivers is low and some make no contribution. The impact of these four rivers is also much higher in the wet and dry season than in the moderate rain season, because long and short rain events occur during these periods.

These insights into the effects of changing inflow and outflows which are based on a range of various data sources are crucial in the context of increased environmental variability due to climate change.

Indeed, cyclic and periodic precipitation events are changing lake inflows and are responsible for increases and decreases in the water level in Lake Victoria. Cyclic and extreme precipitation events seems to be the greatest source of water level rises and these events increase river discharges and associated transport to the lake. These fluxes are likely to increase with increased frequency of future climate change events, such as flood events. While a hydropower dam regulates lake water level and outflow, increased temperature and drought events can lower the water level, making numerical hydro-meteorological analysis important for determining lake water level variability, seasonal effects, extreme events, and for water quality assessments.

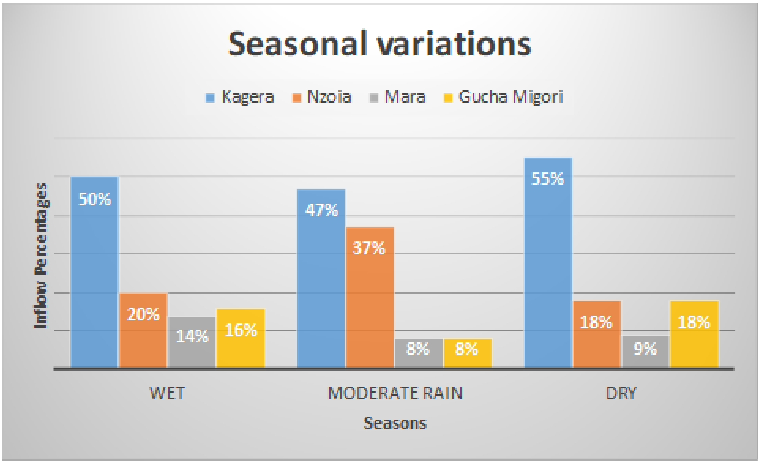

Figure 23: Seasonal high peak (1950-2000) variations in inflow to Lake Victoria from the major tributaries Kagera, Nzoia, Mara, and Gucha-Migori in the wet, moderate rain, and dry periods.

5.2. Modelling pollutants from rivers and wind patterns

To understand the dispersion of pollutants and nutrients is a recurrent and important hydrological problem, especially when modelling shallow lakes under high human use. This thesis showed how pollutants and tracer concentrations can be modelled for shallow lakes (**paper IV and VI**). The advection-dispersion model (**Equation 11**) was employed to assess how a tracer or contaminant moves in the lake, which was used to analyze and illustrate how water from different rivers spread into the lake (remembering also that the outflow of these rivers can change with climate variability). Using empirical data, the thesis could use the model to demonstrate (in **Figure 16**) how the water from Kagera, Nzoia, Mara, and Yala rivers were dispersed from 1950 to 2006.

The wind is however another strong flow velocity factor in water dynamics that will affect pollution movement. Initially, the three Tanzanian weather stations data recorded for 30 years from 1986 – 2016 was used to analyses wind flow velocity and longer time trends (**Figure 24b-d**) and a 20 day wind data set for Entebbe on the Uganda lakeshore was also added (**Figure 24a**). Based on these wind data, a vertically integrated SWEs model was applied to explore the potential role of wind for water and pollution movement for the Murchison Bay surface water (**Paper 4**). This model also included an artificial diffusion under depth-resolved hydrostatic model for pollution transport that could account for transport processes in the lake's hydrodynamics.

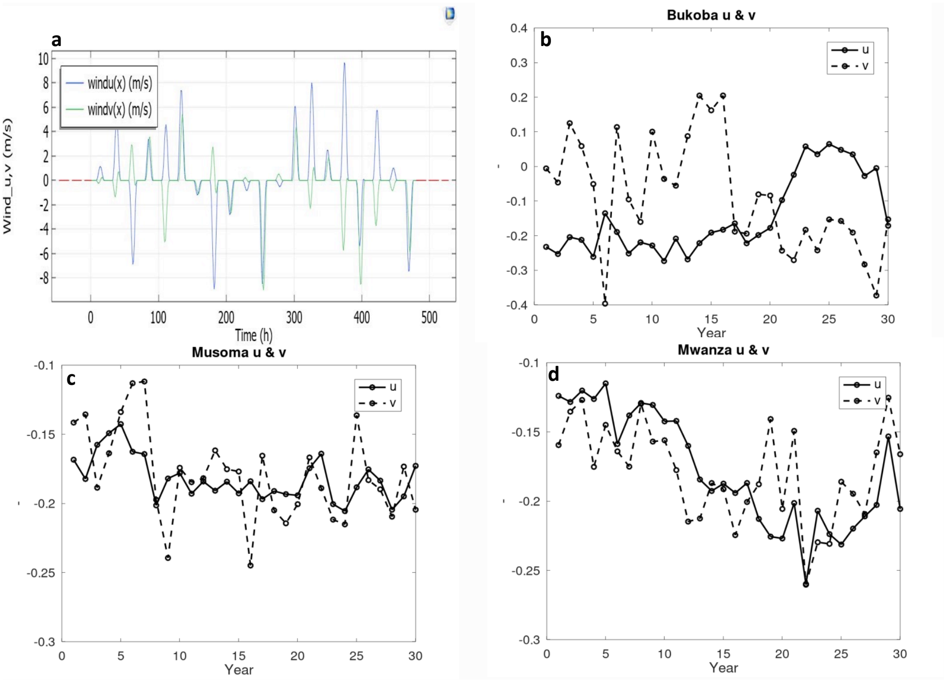

Figure 24: Wind field component analysis of Entebbe and Tanzania three stations, a) Entebbe 480 hrs/ 20 days data, 30 years data b) Bukoba, c) Musoma, and d) Mwanza.

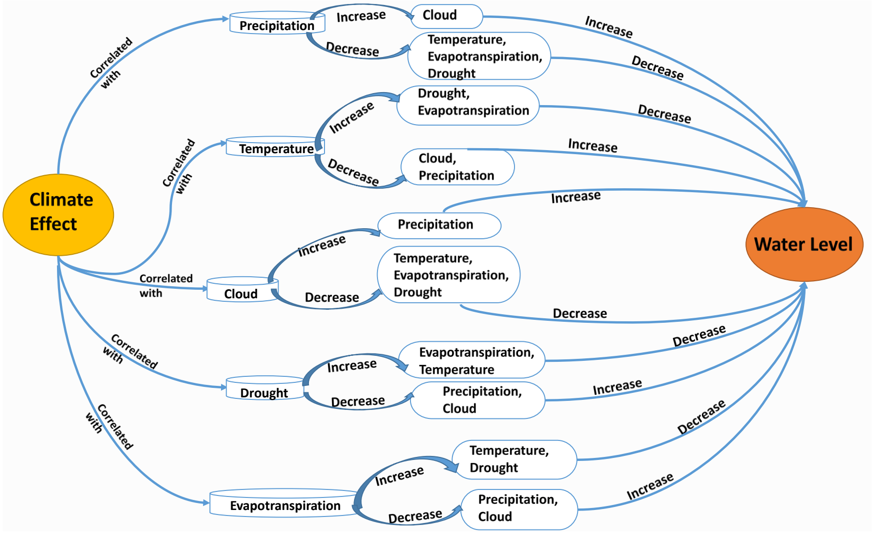

Figure 25: The flowchart shows the interrelation between the effects of climate with lake water levels.

Apart from linking wind with pollution transport, this approach provides an opportunity for long-term wind analysis, which for Lake Victoria demonstrated increasing wind trends in peak time

(May-July, Dec-Feb) and decreasing trends in the off-season, noting an overall upward and downward trend from 1986 to 2016 (Figure 24b-d) with a seasonal wind pattern that could influence the sloshing of the lake. Still, a significant monthly trend was detected on only two stations: Bukoba station (western part of the lake) and Mwanza station (southern part of the lake) that showed a significant upward trend (detailed in **paper IV**).

Results from analysing how wind relates to pollution transport (**paper IV**) suggested that the Murchison Bay wind stress would need to move the whole water column and that would occur much too slowly for the wind to be a dominant factor in the transport of pollutants from MB into the wider Lake Victoria water body. However, it was also discovered that the depth-resolved hydrostatic model considered required artificial dissipation for stability. Based on the wind pattern analysis and modelling results, it appears that full Navier-Stokes equations resolved with depth would be required to accurately model the role of wind on hydrodynamics and thus on pollution transport. So, while the approach discussed here starts to directly link wind and pollution transport, further analysis would be needed to ascertain the true role of wind on mixing and transport in MB and Lake Victoria.

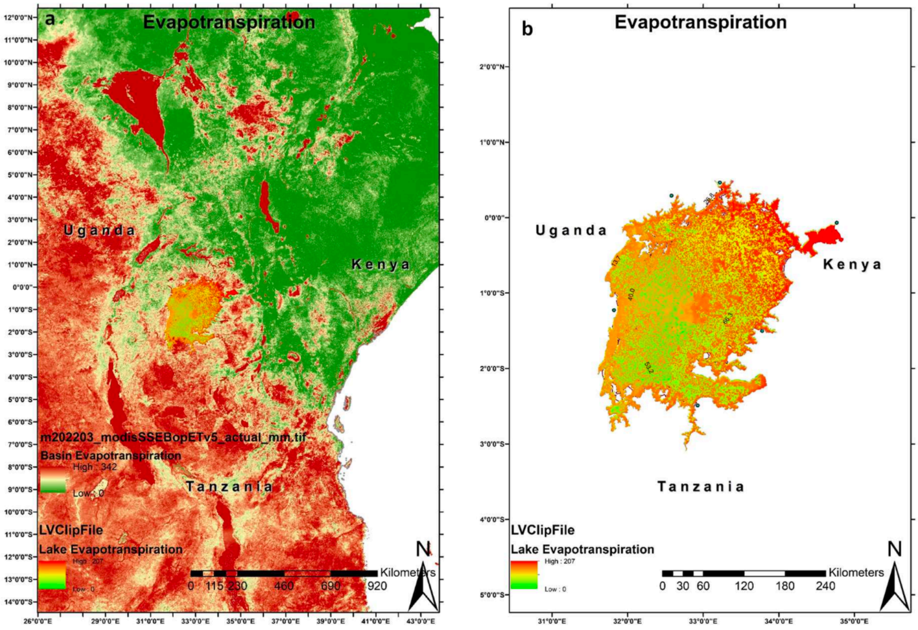

Figure 26: Evapotranspiration effect on Lake Victoria, a) Evapotranspiration on surrounding lake basin, b) evapotranspiration is affecting on Lake Victoria.

5.3 Towards integrating catchment data to account for climate change

The numerical hydrological model implemented in Comsol and that sits at the heart of this thesis has opened the possibility to build more integrative climate change models for Lake Victoria at the catchment scale. Several studies have already examined the impact of climate change on rainfall patterns and lake levels of Lake Victoria. For instance, Akurut et al. (2014) focused on how climate change may alter rainfall and evaporation patterns. Tungaraza et al. (2012) highlighted the long-term climate impact on lake level fluctuations and resource availability. In 2019 and 2020, Khaki and Awange (2021) observed that lake levels rose at an alarming rate during two major events occurring between April and May 2019, and December 2019 and January 2020.

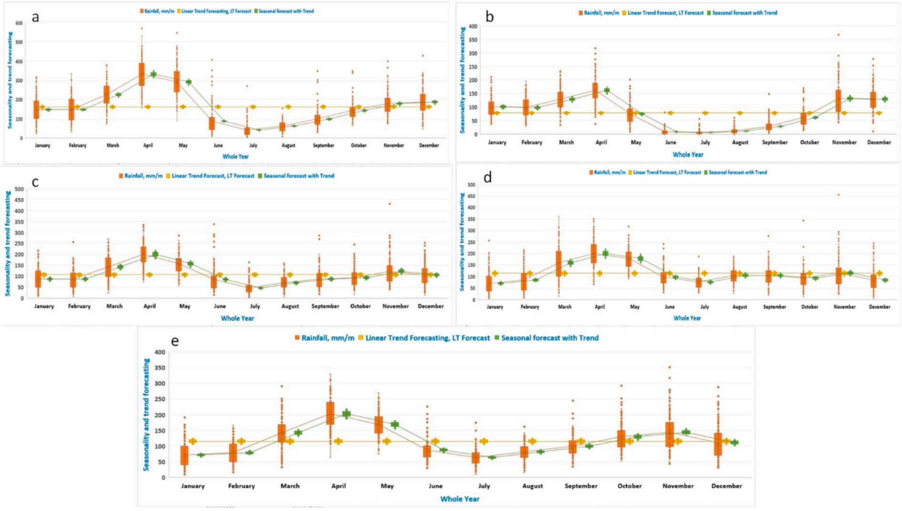

Figure 27: The long-time data set (1900-2020) for forecasting both trend and seasonality, a) Bukoba rainfall trend, b) Mwanza rainfall trend, c) Musoma rainfall trend, d) Kisumu rainfall trend, e) Entebbe rainfall trend.

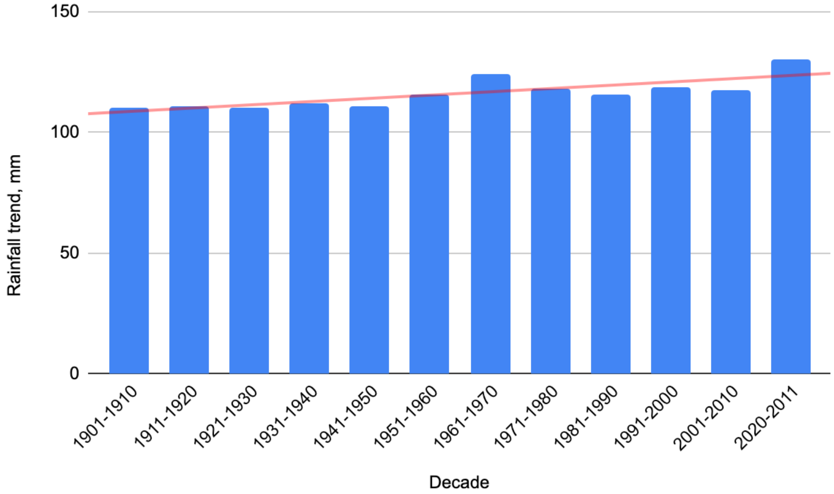

Figure 28: The mean annual rainfall trend per decade from 1900 to 2020 with trendline showing steady increase of mean annual rainfall.

Drawing on theses scholars and others, **paper VI** developed a more integrative approach to account for how Lake Victoria is influenced by climate variability measured as long-term data series at the catchment scale (**Chapter 4.10, Figure 22**). The underlying framework for this integration is seen in **Figure 25**, outlining factors that link climate change (as driver) with water levels (being modelled), including precipitation, temperature, cloud cover, drought, and evapotranspiration. I have added figures on evapotranspiration and rainfall trends from **paper VI** to illustrate the integrative approach.

First, NASA/EOS datasets are useful in bringing evapotranspiration into the fold. For Lake Victoria it was possible to estimate that the Kenya lakeshore evapotranspiration is much higher than Uganda and Tanzania (**Figure 26**). Tanzania's land surface evapotranspiration rate is furthermore much higher than in Uganda and Kenya because there is uncontrolled loss of vegetation, less air moisture, and a rainfall rate lower than in other lake regions. Uganda and Kenya's far land surface evapotranspiration rates are also lower than the near lake regions and the evapotranspiration rate is lower in the deeper lake zone because density stratification occurs from surface to bottom of the lake. It was additionally observed that the equatorial lakes region has very high values of actual evapotranspiration rate.

Second, using a 120-year-long rainfall data (1900-2020), meteorological trends at the catchment scale can be accounted for, which is directly linked to climate change (**Figure 27**). Here it was clear that the lake has a sub-basin bimodal rainfall pattern with a dry and wet season, with higher rainfall on the western lake side compared to the eastern side and with the southern part of the lake showing decreasing patterns of rainfall compared to the western and northern parts of the lake. Overall, total rainfall is increasing over the 120-year period (**Figure 28**).

The resulting model, where evapotranspiration and rainfall data could be integrated (**Paper 6**), demonstrated a crucial interdependence between hydrodynamic processes and climatological factors with long-term rainfall variation showing a linear positive correlation, which explains the increase in the lake's overall water level (from the section above). With respect to the bathymetry as well as the areal coverage, the presence of nutrients in the lake water could help to identify pollution patterns and water quality, which are all crucial factors in determining water levels and water quality, especially in the face of climate change. Figure 28 tries to summarize the integrative potential of the hydrodynamic Comsol model, where a range of factors of how climate change influence lake hydrodynamics are outlined.

6. Conclusion

The thesis was designed to understand shallow lake hydrodynamics with the aim to develop a process for how to develop a detailed bathymetry mapping, a lake flow model and its verification and validation with lake water level as well as tracer and pollution transport processes, and analysis of wind patterns, catchments discharge, and climate effects. The research was carried out through the case study of Lake Victoria, one of the largest and most important freshwater lakes in the world and where large pollution problems in near lakeshore areas impact millions of people that depend on lake water for their livelihood. Long-term remote-sensing and observational datasets were used to visualize the results in geographical information system (GIS) and Comsol models.

Several methods and approaches were tested to set up a lake bathymetry model together with hydrodynamic flow transport for Lake Victoria. 2D shallow water equations have been developed for a free lake surface flow relevant for lake numerical and hydrodynamic modelling. The comparison with measured water levels to provide model verification and the development of a modelling framework was validated using measurements of mean water level. Rainfall, which is the most influential parameter on lake flow, both through direct precipitation over the lake and over the catcment area seen in increased river discharge, has a strong seasonal variation. There is a strong correlation between the tributary inflows and precipitation, and between lake outflow and water level. The advection-dispersion development model integrated with a vertically integrated SWEs model shows that the tracer transport is associated with water circulation effects which leads to that solute transport moves slowly in the lake.

A wind induced lake hydrodynamics model focuses on the wind-driven flow in Lake Victoria as an extremely shallow and geometrically complex lake with islands and bays. Measurements and a numerical model that was developed for Lake Victoria and in more detail for the Murchison Bay (MB) focused on nearshore phenomena characterized through wind stress hydrodynamics and pollution transport process. The MB wind stress would need to move the whole water column and a steady wind have little influence on the horizontal water exchange. It was discovered that the depth-resolved numerical hydrostatic numerical model required artificial dissipation for analyses of pollution transport processes. Finally, the lake's flow, water level and effects from climate change are affected by long-term rainfall patterns which also influence pollutant patterns and the quality of the lake water at different depths and areas of the lake.

The following specific conclusions are drawn from the development and use of the model:

1. The systematically implemented lake bathymetry mapping and the vertically integrated Saint-Venant SWE models implemented for lake flow behaviour and a linear outflow boundary condition achieved a significant correlation with measured outflow. Computed water level agreed reasonably with measurements, which is crucial for lake numerical model validation. The research can be used for bathymetry development, lake flow control, and crucial monitoring of lake water level.

2. The hydro-meteorological processes are influencing lake water balance and the model calculation had reasonable agreement with measured data. The research showed strong correlation between tributary inflows and precipitation, and between outflow and lake water level. The research also showed how an advection-dispersion tracer model can assess basic transport patterns in the lake, in particular tracing how pollution moves through the lake or where they reside or settle in the lake.

3. SWEs were chosen as a primary test for developing a first suit of hydrodynamic models of Lake Victoria. The surface flow model developed showed that the prevailing wind moves from southeast to northwest and phosphorous deposition is affecting Lake Victoria.

4. The vertically integrated sloshing model (used when modelling Murchison Bay) shows the minimal wind effect on lakeshore regions. The thesis discovered that the depth-resolved hydrostatic equations required artificial dissipation to move pollution from shallower lake zone to deeper lake areas. The research can be used for pollution transport processes and risk assessment, which are of great importance to control water quality.

5. The model developed for falling and moving "wiggling" objects can be used for studies of the small and slow moving of microorganism in viscous fluid or polluted water.

6. The study has revealed some of the climate change effects on lake dynamic processes and how they interact with hydrodynamic and nutrient processes in the lake water systems. The development of a hydrodynamic model for Lake Victoria has increased the potential to better understand climate change effects at the catchment scale of this important economic

and ecological region of Africa. The research can provide knowledge to improve climate risk assessments in lake areas.

An exciting pathway for future studies would be to combine computational models of chemical, biological and ecological systems relevant for the lake with a groundwater and a surface water system model to assess water quality. The nutrient and pollution analyses that is part of the current model developed in this thesis could be further developed by integrating more data on source of pollution, rural and urban effluents, rainfall and river runoff, sediment, and chemical transport, and finally heat and mass transport. A eutrophication model could be based on flow related variables of pollution load measurements, nutrient loading, algae blooming, and phytoplankton dispersal. The improvement of such a water quality model could be adapted to give a general computational simulation model with emphasis on water systems such as aquifers, lakes, and rivers.

The future research modelling activities could be summarized as below:

1. Investigate water quality models using 3D models with density stratification on vertical velocity profiles. Such 3D simulation results could support 2D existing models by e.g., tuning the turbulence models for mixing and pollution transport in the water exchange.

2. Measure and assess the amounts of pollution and nutrients that are entering the lake and develop biophysical and biological models to better understand the role of eutrophication on water quality in different areas of the lake.

The existing numerical hydrodynamic model that was built on the COMSOL Multiphysics software and that lies at the heart of this thesis is already useful and forms a base for the above-mentioned future research. It can be used for further assessing the effects of deterioration of water quality, provide insight into the dominant pollutant pathways and eutrophication of Lake Victoria and form the basis for defining most effective protection and management strategies. In addition, there is hope that this study could help increase local capacity in water resources, ecosystem services and sustainable development at significant regional knowledge centres that also contributed to this study, including the Makerere University and National Water Sewage Corporation in Uganda, the Kenyatta University and Lake Victoria Basin Commission in Kenya and the Dar es Salaam University, Water Ministry and The Meteorological Agency in Tanzania, as well as other organizations around the lake basin.

Taken together, this dissertation has contributed new insights into lake hydrodynamic processes and pollution transport in shallow lakes through developing more accurate models to understand the complex processes of water quality degradation, which can be applied in research across the world that is interested in combining empirical data and advanced mathematical modelling. Indeed, the thesis illustrated the power of numerical and hydrodynamical methods that uses one- and two-dimensional mathematical equations to model the three-dimensional behaviour of shallow lakes over time. Such fully developed lake hydrodynamic models as they are developed for more lakes and water bodies across the world, and validated against empirical measurements, could form a formidable foundation for how to manage scarce freshwater bodies in a world of local to planetary environmental change. This thesis, which has used hydrology and advanced mathematics to get to know *Nalubaale*, the “abode of the spirits,” is but a humble contribution to such an international and very much needed effort.

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